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Structure constants of planar $\mathcal{N}=4$ Yang Mills at one loop

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Abstract: We study structure constants of gauge invariant operators in planar $\mathcal{N}=4$ Yang-Mills at one loop with the motivation of determining features of the string dual of weak coupling Yang-Mills. We derive a simple renormalization group invariant formula characterizing the corrections to structure constants of any primary operator in the planar limit. Applying this to the scalar $SO(6)$ sector we find that the one loop corrections to structure constants of gauge invariant operators is determined by the one loop anomalous dimension Hamiltonian in this sector. We then evaluate the one loop corrections to structure constants for scalars with arbitrary number of derivatives in a given holomorphic direction. We find that the corrections can be characterized by suitable derivatives on the four point tree function of a massless scalar with quartic coupling. We show that individual diagrams violating conformal invariance can be combined together to restore it using a linear inhomogeneous partial differential equation satisfied by this function.

Contents

1. Introduction

By far, the most precise realization of field theories being dual to string theories occurs in examples of the AdS/CFT correspondence proposed by Maldacena [1, 2, 3]. Among these examples, the most studied case is the duality between $\mathcal{N} = 4$ Yang-Mills theory in four dimensions with gauge group $U(N)$ and type IIB string theory on $AdS_5 \times S^5$. Let us briefly recall the map between the basic parameters of the string theory and $\mathcal{N} = 4$ Yang-Mills. It is convenient to set the radius of AdS to one

so that in such units the string length is related to the t'Hooft coupling of the gauge theory by

$$\alpha' = \frac{1}{\sqrt{\lambda}} = \frac{1}{\sqrt{g_{YM}^2 N}}, \qquad G_N = \frac{1}{N^2}, \tag{1.1}$$

here g_{YM} is the Yang-Mills coupling constant, α' refers to the string length and G_N is the Newton's constant in these units which is the effective string loop counting parameter.

The regime in which this duality has been mostly explored is when the type IIB string theory can be approximated by type IIB supergravity. To decouple all the string modes, the t'Hooft coupling has to be large. Furthermore, to suppress string loops we need to work at large N. One can then set up a precise correspondence of gauge invariant operators and supergravity fields. Another interesting limit, which has received a lot of attention recently, is when the t'Hooft coupling λ is small but with N still being large. In this limit especially when λ is strictly zero, all string modes are equally important but string loops are suppressed. From (1.1) we see that λ being zero implies the string length is infinity, the $AdS_5 \times S^5$ string sigma model is strongly coupled. At present there are no known methods to extract any information regarding the spectrum or the correlation functions from the strongly coupled sigma model. On the other hand, the dual field theory is best understood in this limit since at $\lambda = 0$ the theory is free and planar perturbation theory in the t'Hooft coupling is sufficiently easy to perform. This has led to many efforts in trying to rewrite the spectrum of the $\mathcal{N} = 4$ Yang-Mills theory as a spectrum in a string theory [4, 5, 6]. There has also been an effort at reconstructing the string theory world sheet by rewriting the correlation function of gauge invariant operators of the free theory as amplitudes in AdS [7, 8].

In this paper, with the motivation to find features of the string theory at weak coupling Yang-Mills we study structure constants of certain class of gauge invariant operators in planar $\mathcal{N} = 4$ super Yang-Mills, at one loop in t'Hooft coupling. To indicate which features of the string theory one would expect to see by studying the structure constants, we first need to provide the picture of the string theory at $\lambda = 0$ limit that we have in mind. From (1.1) we see that at $\lambda = 0$ the string essentially becomes tensionless, therefore there is no coupling between neighboring points on

the string which breaks up into non interacting bits. In fact this picture of the string has already been noticed in the plane wave limit [9] and has been discussed in the context of string theory in small radius AdS [10]. From studies of correlation functions of gauge invariant operators in the plane wave limit, it is seen that each Yang-Mills letter can be thought of as a bit in a light cone gauge fixed string theory, and a single trace gauge invariant operator is a sequence of bits with cyclic symmetry [11, 12, 13, 14, 15] A universal feature of any string field theory is that interactions are described by delta function overlap of strings. Therefore the structure constants of gauge invariant operators, which in the planar limit are proportional to $1/N$, should be seen as joining or splitting of strings. Indeed, it is possible to formulate a bit string theory in which all features of the two point functions and structure constants of gauge invariant operators, including position dependence, can be reproduced by the delta function overlap [16].

Now let us ask the question of what would be the modifications in the above picture when one makes λ finite. From (1.1) we see that rendering α' finite would introduce interactions between the bits. At first order in λ and in the planar limit, only nearest neighbor bits would interact. Therefore, turning on λ modifies the free propagation of the bits in the bit string theory. The one loop corrected two point function and the structure constants should still be determined by the geometric delta function overlap, but with the modification in the propagation of the bits taken into account. Thus identifying the precise operator which is responsible for the propagation of the bits at first order in λ, should be sufficient to determine the modified two point functions and the structure constants at one loop. It is this feature of Yang-Mills theory we hope to uncover by studying the structure constants.

Apart from the above motivations, from a purely field theoretic point of view a conformal field theory is completely specified by the the two point functions and the structure constants of the operators. A lot of effort have been made to understand the structure of the two point functions of gauge invariant operators of $\mathcal{N}=4$ Yang-Mills in the planar limit. In fact the anomalous dimension Hamiltonian at one loop in λ is known to be integrable [17, 18, 19, 20], and signatures of integrability in the form of the existence of an infinite number of nonlocal conserved charges has been

shown for the world sheet theory on $AdS_5 \times S^5$ [21, 22, 23, 24, 25]. Furthermore, the relation between these approaches to integrabilty have been studied in [26, 27, 28]. On the other hand structure constants of operators in $\mathcal{N} = 4$ theory are considerably less explored [29, 30, 31]. One difficulty in studying corrections to structure constants is that one needs to find the right renormalization group invariant quantity which characterizes the corrections

In this paper we derive a simple formula which characterizes the renormalization group invariant quantity which determines the corrections to structure constants of primary gauge invariant operators. Then we use this to study the one loop corrections to structure constants in the scalar $SO(6)$ sector and a sector of operators with derivatives in a given holomorphic direction. We find that in the $SO(6)$ sector the renormalization invariant quantity, which determines the one loop correction to the structure constants, is the one loop anomalous dimension Hamiltonian itself. Evaluation of the structure constants for operators with derivatives is considerably more involved. Feynman graphs contributing to the corrections can be obtained by a suitable combination of derivatives acting on the function $\phi(r, s)$, which refers to the tree level four point function of a massless scalar with a quartic coupling and r, s are the two conformal cross ratios. There are individual Feynman diagrams contributing to the one loop corrections to structure constants which seem at first to violate conformal invariance, but we find that the violating diagrams can be combined together using the fact that $\phi(r, s)$ satisfies a linear inhomogeneous partial differential equation ensuring conformal invariance [1].

This paper is organized as follows. In section 2. we derive the renormalization group invariant formula characterizing the corrections to structure constants of primary operators. In section 3. we apply this to the scalar $SO(6)$ sector and show that corrections are captured by the one loop anomalous dimension Hamiltonian. The fact that the anomalous dimension Hamiltonian captures the correction to the structure constants was observed in [30]. Their observation relied on certain examples and the statement that only the F terms occur in the Feynman diagrams. The proof given here is direct and the method is suitable for extension for classes of operators

[1] After completion of this work it was pointed out to us by G. Arutyunov, that similar differential equations have been studied in [32, 33]

in other sectors. In section 4. we compute the corrections to structure constants for operators with derivatives in one holomorphic direction. We show that conformal invariance in the three point function is ensured by the differential equation satisfied by $\phi(r,s)$. The summary of the results which enables one to calculate the structure constants to any operator in this sector is given in section 4.4. Appendix A. contains the notations adopted in the paper, Appendix B discusses the properties of the function $\phi(r,s)$, in particular it contains the proof of the differential equation it satisfies. Appendix C. contains tables which are required in the evaluation of the structure constants in the derivative sector.

2. General form of structure constants at one loop

Our aim in this section is to derive a formula which gives a renormalization group invariant characterization of one loop corrections to structure constants at large N. Consider a set of conformal primary operators labelled by $O_i^{\mu_1\ldots\mu_{n_i}}$, here $\mu_1\ldots\mu_{n_i}$ indicate the tensor structure of the primary [2]. For simplicity, let us suppose the basis of operators is such that their one loop anomalous dimension matrix is diagonal, we will relax this assumption later. Then, by conformal invariance, the general form for the two point function of these operators at one loop is given by:

$$\langle O_i^{\mu_1\ldots\mu_{n_i}}(x_1)O_j^{\nu_1\ldots\nu_{n_j}}(x_2)\rangle = \frac{J^{\mu_1\ldots\mu_{n_i};\nu_1\ldots\nu_{n_i}}}{(x_1-x_2)^{2\Delta_i}}\left(\delta_{ij}+\lambda g_{ij}-\lambda\gamma_i\delta_{ij}\ln((x_1-x_2)^2\Lambda^2)\right). \tag{2.1}$$

Here $J^{\mu_1\ldots\mu_{n_i};\nu_1\ldots\nu_{n_i}}$ is the invariant tensor constrained by conformal invariance and constructed by products of the following tensor:

$$J^{\mu\nu} = \delta_{\mu\nu} - 2\frac{(x_1-x_2)^\mu(x_1-x_2)^\nu}{(x_1-x_2)^2}. \tag{2.2}$$

Since we are interested in the one loop correction in the planar limit, the expansion parameter in (2.1) $\lambda = g_{YM}^2 N/32\pi^2$ is the t' Hooft coupling. In (2.1) we have used the fact that it is possible to choose a basis of operators such that they are orthonormalized at tree level and that their anomalous dimension matrix is diagonal. Δ_i are the bare dimensions and γ_i refer to the anomalous dimensions of the respective

[2] In this paper will restrict our attention to primaries which are tensors, but our methods can be generalized to other classes of operators.

operators. For non zero tree level two point function in (2.1) $\Delta_i = \Delta_j$ and $n_i = n_j$. The constant mixing matrix at one loop g_{ij} is renormalization group scheme dependent, for instance if the cut off Λ is scaled to $e^{\alpha}\Lambda$, the mixing matrix changes as follows:

$$g_{ij} \rightarrow g_{ij} - 2\alpha\gamma_i\delta_{ij}. \tag{2.3}$$

The three point function of three tensor primaries is given by:

$$\begin{aligned} \langle \quad & O_i^{\mu_1\ldots\mu_{n_i}}(x_1) O_j^{\nu_1\ldots\nu_{n_j}}(x_2) O_k^{\rho_1\ldots\rho_{n_k}}(x_3)\rangle \qquad (2.4)\\ = & \frac{J^{\mu_1\ldots\mu_{n_i};\nu_1\ldots\nu_{n_j};\rho_1\ldots\rho_{n_k}}}{|x_{12}|^{\Delta_i+\Delta_j-\Delta_k}|x_{13}|^{\Delta_i+\Delta_k-\Delta_j}|x_{23}|^{\Delta_j+\Delta_k-\Delta_i}} \times \\ & \left(C_{ijk}^{(0)} \left[1 - \lambda\gamma_i \ln|\frac{x_{12}x_{13}\Lambda}{x_{23}}| - \lambda\gamma_j \ln|\frac{x_{12}x_{23}\Lambda}{x_{13}}| - \lambda\gamma_k \ln|\frac{x_{13}x_{23}\Lambda}{x_{12}}| \right] + \lambda \tilde{C}_{ijk}^{(1)} \right), \end{aligned}$$

where $x_{12} = x_1 - x_2, x_{13} = x_1 - x_3, x_{23} = x_2 - x_3$. Note, that from large N counting it is easy to see that both $C_{ijk}^{(0)}$ and the one loop correction $\tilde{C}_{ijk}^{(1)}$ are order $1/N$. Again the constant one loop correction to the $\tilde{C}_{ijk}^{(1)}$ is renormalization scheme dependent, scaling Λ by $e^{\alpha}\Lambda$, we see that:

$$\tilde{C}_{ijk}^{(1)} \rightarrow \tilde{C}_{ijk}^{(1)} - \alpha \left(\gamma_i C_{ijk}^{(0)} + \gamma_j C_{ijk}^{(0)} + \gamma_k C_{ijk}^{(0)} \right). \tag{2.5}$$

Here there is no summation of repeated indices. Therefore from (2.3) and (2.5) we see that the following combination is renormalization scheme independent

$$C_{ijk}^{(1)} = \tilde{C}_{ijk}^{(1)} - \frac{1}{2} g_{ii'} C_{i'jk}^{(0)} - \frac{1}{2} g_{jj'} C_{ij'k}^{(0)} - \frac{1}{2} g_{kk'} C_{ijk'}^{(0)}, \tag{2.6}$$

where summation over the primed indices is implied. Essentially, the renormalization scheme independent one loop correction to the structure constant is obtained by first normalizing all the two point function to order λ. We now write the equation (2.6) using an arbitrary basis of primaries. Let the transformation matrix which takes the orthonormalized basis of primaries to an arbitrary basis, be given by $U_{\alpha i}$, where $\alpha, \beta \ldots$ label the arbitrary basis, of primaries. This transformation is λ independent since it is possible to choose a basis of operators which are orthonormalized at tree level and their one loop anomalous dimension matrix is diagonal. The transformation matrix $U_{\alpha i}$ satisfies the following relations:

$$\sum_i U_{\alpha i} U_{\beta i} = h_{\alpha\beta}, \qquad \sum_i U_{\alpha i}\gamma_i U_{\beta i} = \gamma_{\alpha\beta}. \tag{2.7}$$

Here $h_{\alpha\beta}$ is the tree level mixing matrix and $\gamma_{\alpha\beta}$ is the anomalous dimension matrix at one loop. It is usually convenient to chose a basis with $h_{\alpha\beta} = \delta_{\alpha\beta}$, in standard literature the anomalous dimension matrix is specified in such a basis. But here we will work with an arbitrary basis, performing change of basis in (2.6) we obtain:

$$C^{(1)}_{\alpha\beta\gamma} = \tilde{C}^{(1)}_{\alpha\beta\gamma} - \frac{1}{2} g_{\alpha\alpha'} C^{(0)\alpha'}_{\ \ \ \ \beta\gamma} - \frac{1}{2} g_{\beta\beta'} C^{(0)\ \beta'}_{\alpha\ \ \ \gamma} - \frac{1}{2} g_{\gamma\gamma'} C^{(0)\ \ \ \gamma'}_{\alpha\beta}, \tag{2.8}$$

where:

$$\begin{aligned}
\tilde{C}^{(1)}_{\alpha\beta\gamma} &= U_{\alpha i} U_{\beta j} U_{\gamma k} \tilde{C}^{(1)}_{ijk}, \qquad C^{(0)}_{\alpha\beta\gamma} = U_{\alpha i} U_{\beta j} U_{\gamma k} \tilde{C}^{(0)}_{ijk}, \\
C^{(0)\alpha}_{\ \ \ \ \beta\gamma} &= h^{\alpha\alpha'} C^{(0)}_{\alpha'\beta\gamma}, \qquad C^{(0)\ \beta}_{\alpha\ \ \ \gamma} = h^{\beta\beta'} C^{(0)}_{\alpha\beta'\gamma}, \quad C^{(0)\ \ \ \gamma}_{\alpha\beta} = h^{\gamma\gamma'} C^{(0)}_{\alpha\beta\gamma'}, \\
h^{\alpha\alpha'} h_{\alpha'\beta} &= \delta^{\alpha}_{\beta}.
\end{aligned} \tag{2.9}$$

We will call the subtractions in (2.8) as metric subtractions.

2.1 The slicing argument

We work towards a useful characterization of the formula given in (2.8). Local gauge invariant operators can be constructed by products of the fundamental letters of $\mathcal{N} = 4$ Yang Mills and finally taking a trace. We represent a general Yang Mills letter by W^A, then a gauge invariant operator is $\mathrm{Tr(W^A W^B \cdots W^Z)}$. The tree level contractions which contribute to $C^{(0)}_{\alpha\beta\gamma}$ of three gauge invariant primaries at the planar level are all possible Wick contractions which can be drawn on a plane using the double line notation. We can represent a given contraction by the diagram in fig. 1, the corresponding double line notation is given adjacent to it. In fig. 1 we have used single lines to represent the double line. The lines end on letters of the operators, these are points on the horizontal lines in the diagram.

Consider the one loop correction $\tilde{C}^{(1)}_{\alpha\beta\gamma}$, contributions to this can arise from two types of terms: (i) two body terms represented by $U_{\alpha\beta}, U_{\alpha\gamma}$ and $U_{\beta\gamma}$ in fig. 2 (ii) genuine three body terms represented by $U^{\alpha}_{\beta\gamma}, U^{\beta}_{\gamma\alpha}, U^{\gamma}_{\alpha\beta}$ as shown in fig. 3. As we are interested in planar corrections at one loop, it is easy to see that the two body interactions can occur only between nearest neighbour letters of any two of the operators with the remaining contractions performed at the free level. There is an exception to this rule, when the structure constant of interest is length conserving, for instance when say, the length of operator O_α equals the sum of the lengths of the

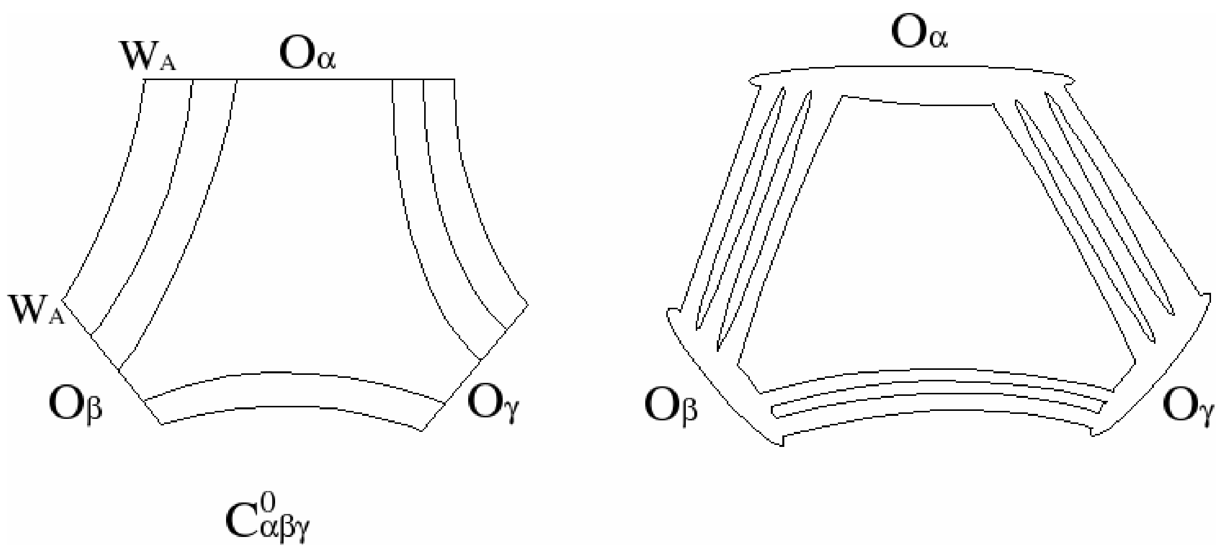

Figure 1: Planar Wick contractions contributing to $C^{(0)}_{\alpha\beta\gamma}$

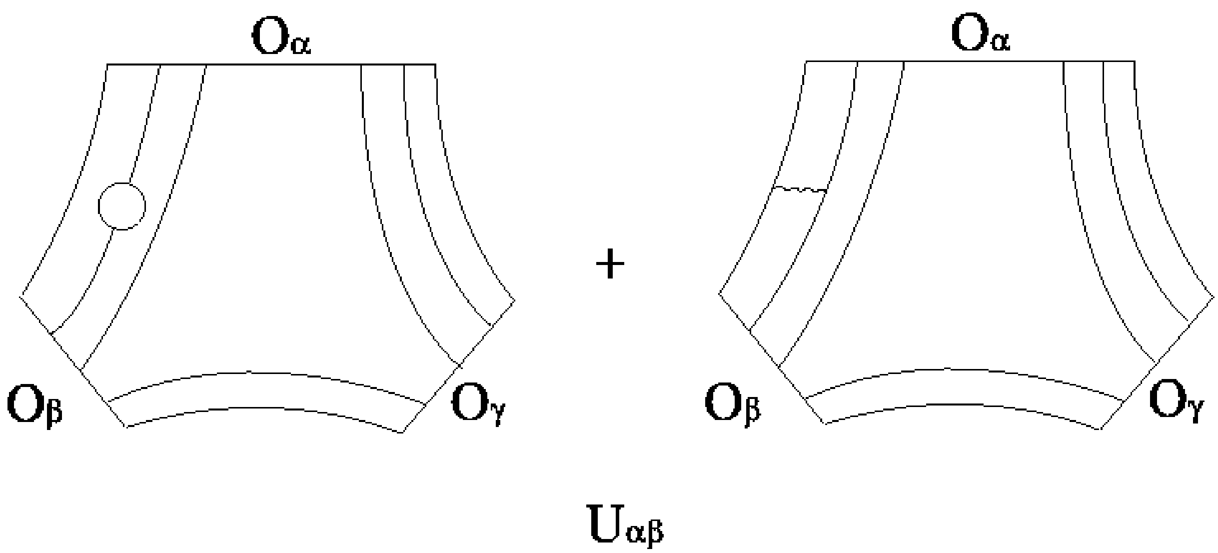

Figure 2: A generic diagram contributing to $U_{\alpha\beta}$

operators O_β and O_γ. We will discuss this case later in the paper, but for now and for most of the discussions in this paper we assume that the structure constants of interest are length non-conserving. Two body interactions can also consist of planar self energy interactions between letters of any two different operators, and the rest of the operators contracted with free Wick contractions. Thus $U_{\alpha\beta}$ represents the sum of all the constants due to all possible nearest neigbour interactions among operators

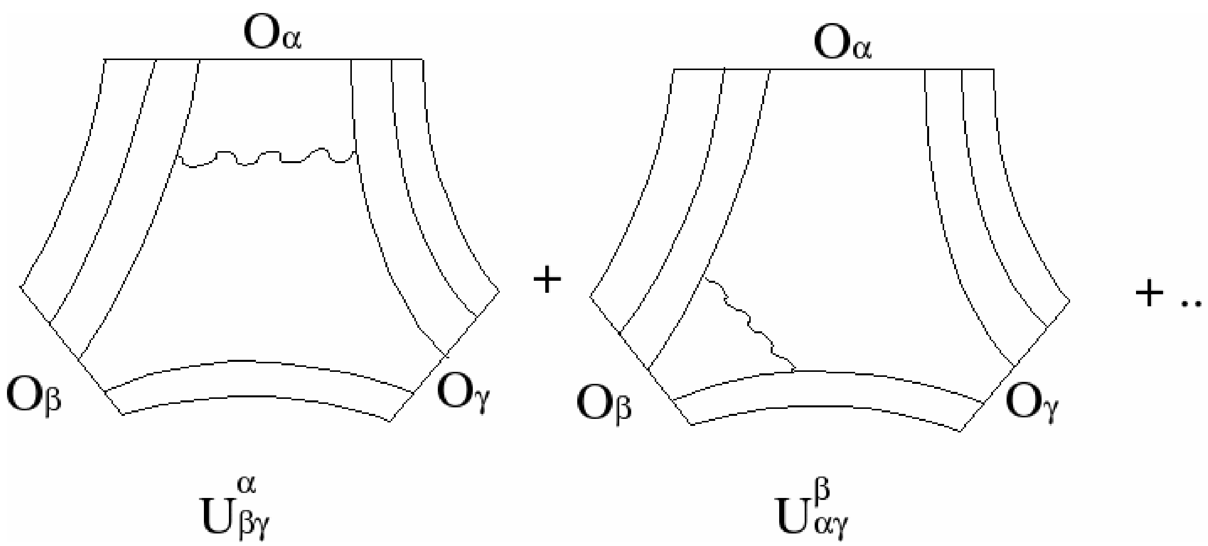

Figure 3: Diagrams contributing to $U^{\alpha}_{\beta\gamma}$ and $U^{\beta}_{\alpha\gamma}$

O_α and O_β, and all possible constants from the self energy interactions between letters of these operators. A similar definition holds for $U_{\alpha\gamma}$ and $U_{\beta\gamma}$. The genuine three body term $U^{\alpha}_{\beta\gamma}$ consists of constants from all possible interactions between any two nearest neighbour letters of the operator O_α and two letters of operators O_β and O_γ such that all contractions are planar. An example of such an interactions are shown in fig. 3. It is easy to see from this diagram that one is forced to choose nearest neighbour letters in operator O_α to ensure that the interaction is planar. Similar definitions hold for $U^{\beta}_{\gamma\alpha}, U^{\gamma}_{\alpha\beta}$. From these definitions we have:

$$\tilde{C}^{(1)}_{\alpha\beta\gamma} = U^{\alpha}_{\beta\gamma} + U^{\beta}_{\gamma\alpha} + U^{\gamma}_{\alpha\beta} + U_{\alpha\beta} + U_{\beta\gamma} + U_{\gamma\alpha}. \tag{2.10}$$

We show now that the two body terms of $\tilde{C}^{(1)}_{\alpha\beta\gamma}$ cancel with the metric subtractions in the equation (2.8). Consider a generic two body interaction in $U_{\alpha\beta}$, imagine slicing the diagram as in fig. 4 by inserting a complete set of operators $O_{\alpha'}$. Thus the diagram decomposes into two halves, the upper half which contains the one loop corrections which can now be viewed as contributions to the one loop correction $g_{\alpha\alpha'}$. The lower half which is just the tree level structure constant $C^{(0)\alpha'}_{\beta\gamma}$. From this slicing we see that exactly the same one loop interaction term occurs in $g_{\alpha\alpha'} C^{(0)\alpha'}_{\beta\gamma}$ [3].

[3]In the first diagram in fig. 4 we have shown only one interaction diagram which on slicing gives a contribution to $g_{\alpha\alpha'}$, other contributions to $g_{\alpha\alpha'}$ also comes from interactions in lines running between O_α and O_β in this slicing.

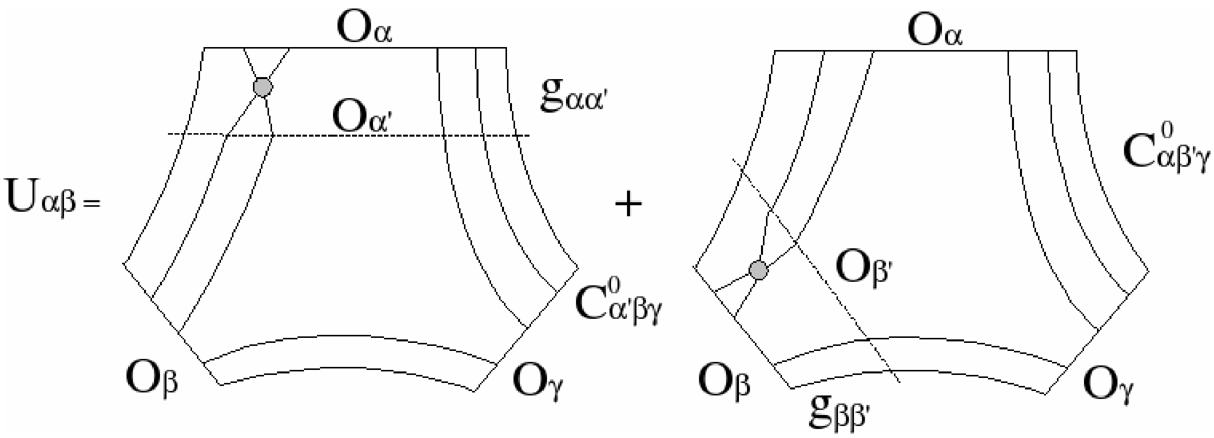

Figure 4: The slicing argument

Now, slice the same diagram as indicated in the second figure of fig. 4 by inserting a complete set of operators $O_{\beta'}$. The one loop correction can be seen as a term in $g_{\beta\beta'}$, while the rest of the diagram as the tree level structure constant $C^{(0)}_{\alpha\ \gamma}{}^{\beta'}$. Thus this diagram also occurs in $g_{\beta\beta'}C^{(0)}_{\alpha\ \gamma}{}^{\beta'}$. In (2.8), the metric subtractions $g_{\alpha\alpha'}C^{(0)\alpha'}_{\beta\gamma}$ and $g_{\beta\beta'}C^{(0)}_{\alpha\ \gamma}{}^{\beta'}$ are weighted by a factor of 1/2, thus we conclude that a generic two body interaction in $U_{\alpha\beta}$ is canceled off by the subtractions in (2.8). This cancellation includes both the nearest neighbour two body interactions as well as the self energy type of interactions which we have not shown in fig. 4. Similar reasoning can be used to conclude that the all the constants in the two body terms $U_{\beta\gamma}$ and $U_{\gamma\alpha}$ also are canceled by the metric subtractions in (2.8).

From the slicing argument we see that the constants from a genuine three body terms in $U^{\alpha}_{\beta\gamma}, U^{\beta}_{\gamma\alpha}, U^{\gamma}_{\alpha\beta}$ cannot be canceled of the metric subtractions. Thus these terms and the corresponding subtraction in (2.8) is what is left behind. This is indicated in the fig. 5. Therefore computation of $C^{(1)}_{\alpha\beta\gamma}$ reduces to the evaluation of constants from diagrams with 4 letters: 2 letters on one operator, say O_α, and the remaining 2 letters on operators O_β and O_γ. From this we subtract half the constants which occur when the same diagram is thought of as the two body interaction, that is 2 letters on one operator say O_α and the remaining 2 letters on the operator O'_α. Summing over all such contributions gives $C^{(1)}_{\alpha\beta\gamma}$. We write this compactly as

$$C^{(1)}_{\alpha\beta\gamma} = \left(U^{\alpha}_{\beta\gamma}(3\text{pt}) - \frac{1}{2}U^{\alpha}_{\beta\gamma}(2\text{pt})\right) + \left(U^{\beta}_{\gamma\alpha}(3\text{pt}) - \frac{1}{2}U^{\beta}_{\gamma\alpha}(2\text{pt})\right) \qquad (2.11)$$

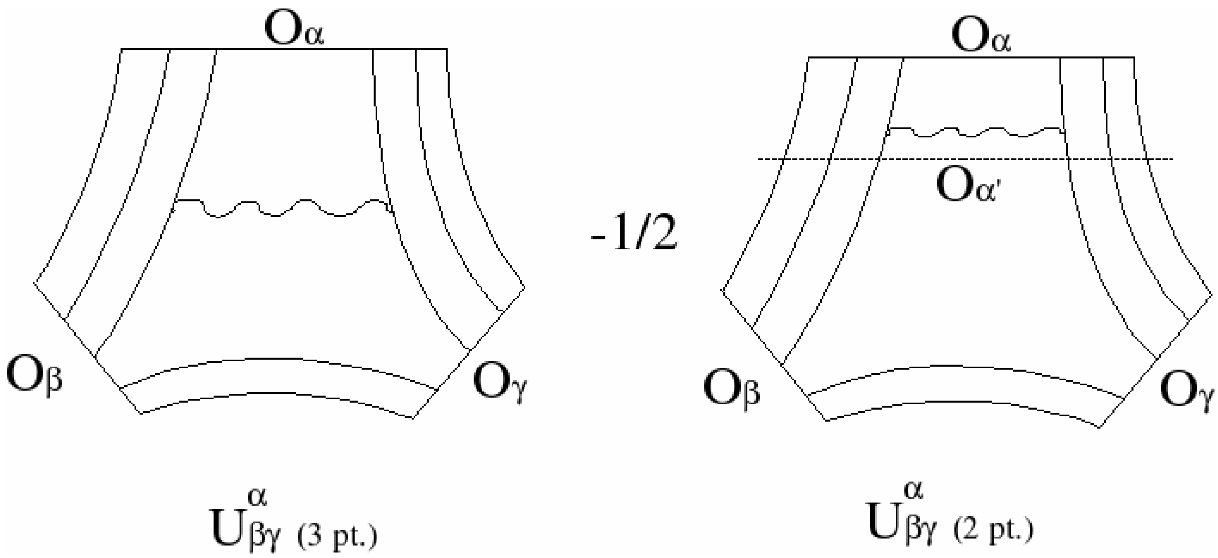

Figure 5: Renormalization scheme independent contribution

$$+\left(U^{\gamma}_{\alpha\beta}(3\text{pt}) - \frac{1}{2}U^{\gamma}_{\alpha\beta}(2\text{pt})\right)$$

Here $U^{\alpha}_{\beta\gamma}(3\text{pt})$ contains constants from genuine three body interactions, that is there are no self energy diagram. $U^{\alpha}_{\beta\gamma}(2\text{pt})$ contains the constants from the same diagrams but now thought of as occurring in a two point function, to emphasize again, this also has no self energy diagrams. Therefore, to compute one loop corrections to structure constants for any arbitrary operator it is sufficient to give the one loop corrections occurring in the computation of any 4 Yang Mills letters, firstly thought of as genuine 3 body interaction and then thought of as a two body interaction.

2.2 An example

We illustrate the slicing argument using a simple example by explicitly evaluating all the terms occurs in (2.8) and showing that it reduces to (2.11). Consider the structure constant when the operators are given by

$$O_\alpha = O_\beta = O_\gamma = \frac{1}{N}\text{Tr}(Z\bar{Z}). \tag{2.12}$$

Here Z is a complex scalar in the one of the Cartan of $SO(6)$, for instance $Z = \frac{1}{\sqrt{2}}(\phi^1 + i\phi^2)$. Thus the Z , $\bar{Z}$ Wick contraction is normalized to 1, which implies that the tree level two point function $h_{\alpha\alpha}$ is normalized to 1. Evaluating the tree level structure constant we obtain $C_{\alpha\alpha\alpha} = 2/N$.

Now consider the one loop corrections to the structure constants. The two body terms consists only of self energy diagrams, these are given by

$$U_{\alpha\beta} + U_{\beta\gamma} + U_{\gamma\alpha} = \frac{\lambda}{N}\left(2S_{\alpha\beta} + 2S_{\alpha\gamma} + 2S_{\beta\gamma}\right) = \frac{\lambda}{N}6S. \tag{2.13}$$

The subscripts in the S are just used to indicate the origin of the constants from the self energy diagrams, for instance there are two self energy diagrams between the Z and $\bar{Z}$ of the O_α and O_β. Since all the self energy diagrams are same they can be summed to give $6S$. We have also kept track of the order of the t' Hooft coupling and N. The genuine three body terms are

$$U^{\alpha}_{\beta\gamma} + U^{\beta}_{\gamma\alpha} + U^{\gamma}_{\alpha\beta} = \frac{\lambda}{N}\left[4H(\alpha;\beta\gamma) + 4H(\beta;\gamma\alpha) + 4H(\gamma;\alpha\beta)\right] = \frac{\lambda}{N}12H(3\mathrm{pt}). \tag{2.14}$$

Here the H basically refers to the constant from the diagram with Z and $\bar{Z}$ on one operator and with $\bar{Z}$ and Z on the remaining two operators. The labels in each of the H just refer to which of the operator has the two letters and which of the rest has a letter each. The factor 4 arises out of the combinatorics of the diagrams. Therefore we have

$$\tilde{C}^{(1)}_{\alpha\alpha\alpha} = \frac{\lambda}{N}\left[6S + 12H(3\mathrm{pt})\right]. \tag{2.15}$$

Now we subtract out the metric contributions in (2.8). We have to sum over all the metric contributions $g_{\alpha\beta'}C^{(0)\beta'}_{\alpha\alpha}$, but this sum reduces to evaluating only one term when $\beta' = \alpha$, this is because all other tree level structure constants vanish. Now $g_{\alpha\alpha}$ is given by

$$g_{\alpha\alpha} = \lambda[2S + 2H(2\mathrm{pt})], \tag{2.16}$$

thus we see that

$$\begin{aligned} C^{(1)}_{\alpha\alpha\alpha} &= \tilde{C}^{(1)}_{\alpha\alpha\alpha} - \frac{1}{2}3g_{\alpha\alpha}C^{(0)\alpha}_{\alpha\alpha}, \\ &= 12\frac{\lambda}{N}\left(H(3\mathrm{pt}) - \frac{1}{2}H(2\mathrm{pt})\right), \end{aligned} \tag{2.17}$$

where we have used (2.15) , (2.16) and substituted the value of $C^{(0)\alpha}_{\alpha\alpha} = h^{\alpha\alpha}C^{(0)}_{\alpha\alpha\alpha} = 2/N$. Note that the self energies which are the only two body terms in $\tilde{C}^{(1)}_{\alpha\alpha\alpha}$ have canceled on subtracting the metric contributions. The last formula in (2.17) is precisely the equation one would have obtained if one uses the formula in (2.11).

3. The scalar $SO(6)$ sector

Consider three operators belonging only to the scalar $SO(6)$ sector given by

$$
\begin{aligned}
O_\alpha &= \frac{1}{N^{l_\alpha/2}}\mathrm{Tr}(\phi^{i_1}\phi^{i_2}\dots\phi^{i_{l_\alpha}}) \\
O_\beta &= \frac{1}{N^{l_\beta/2}}\mathrm{Tr}(\phi^{j_1}\phi^{j_2}\dots\phi^{j_{l_\beta}}) \\
O_\gamma &= \frac{1}{N^{l_\gamma/2}}\mathrm{Tr}(\phi^{k_1}\phi^{k_2}\dots\phi^{k_{l_\alpha}})
\end{aligned}
\tag{3.1}
$$

In this section we show that the renormalization scheme independent correction to the structure constants of this class of operators is essentially dictated by the anomalous dimension Hamiltonian. The invariant one loop correction is given by

$$
C^{(1)}_{\alpha\beta\gamma} = \sum_{a,b,c}\mathcal{H}^{i_a i_{a+1}}_{j_{b+1}k_c}\mathcal{I} + \sum_{a,b,c}\mathcal{H}^{j_b j_{b+1}}_{k_{c+1}i_a}\mathcal{I} + \sum_{a,b,c}\mathcal{H}^{k_c k_{c+1}}_{i_{a+1}j_b}\mathcal{I} \tag{3.2}
$$

where $\mathcal{H}$ is the anomalous dimension Hamiltonian given by [17, 18]

$$
\mathcal{H}^{ij}_{kl} = 2\delta^j_k\delta^i_l - 2\delta^i_k\delta^j_l - \delta^{ij}\delta_{kl}. \tag{3.3}
$$

$\mathcal{I}$ in (3.2) refers to the remaining free planar contractions as shown in fig. 5. The summation over a, b, c runs over all distinct cyclic permutations of the diagram over the indices i, j and k of the three operators. In (3.2) and through out the rest of the paper we will suppressed the λ/N factor which occurs in the normalization of the one loop corrected structure constant.

From the slicing argument it is clear that to show (3.2) one needs to evaluate the following

$$
\begin{aligned}
&\left(U^{i_a i_{a+1}}_{j_{b+1}k_c}(3\mathrm{pt}) - \frac{1}{2}U^{i_a i_{a+1}}_{j_{b+1}k_c}(2\mathrm{pt})\right)\delta^{j_b}_{k_{c+1}} + \left(U^{j_b j_{b+1}}_{k_{c+1}i_a}(3\mathrm{pt}) - \frac{1}{2}U^{j_b j_{b+1}}_{k_{c+1}i_a}(2\mathrm{pt})\right)\delta^{k_c}_{i_{a+1}} \\
&+ \left(U^{k_c k_{c+1}}_{i_{a+1}j_b}(3\mathrm{pt}) - \frac{1}{2}U^{k_c k_{c+1}}_{i_{a+1}j_b}(2\mathrm{pt})\right)\delta^{i_a}_{j_{b+1}}
\end{aligned}
\tag{3.4}
$$

In the above formula $U^{i_a i_{a+1}}_{j_{b+1}k_c}(3\mathrm{pt})$ refers to the constant from the diagram with adjacent letters ϕ^{i_a}, $\phi^{i_{a+1}}$ on the operator O_α and the letters $\phi^{j_{b+1}}$ and ϕ^{k_c} on the operators O_β and O_γ respectively. While $U^{i_a i_{a+1}}_{j_{b+1}k_c}(2\mathrm{pt})$ refers to the constant of the same diagram but thought of as an interaction in a two point calculation. A similar definition holds for the rest of the U's in (3.4). We have written down the Krönecker

delta in each of the terms in (3.4) to denote the adjacent free Wick contractions. The terms in (3.4) are the generic terms that occur when the equation (2.11) is applied to the $SO(6)$ scalars. We will show that after evaluation of the terms in (3.4), the expression reduces to that given in (3.2), essentially the U's are replaced by the anomalous dimension Hamiltonian $\mathcal{H}$.

The claim that the anomalous dimension Hamiltonian dictates the renormalization scheme independent corrections to the structure constants might at first be puzzling to the reader. The anomalous dimension Hamiltonian arises after including self energy diagrams [17, 18] but as we have emphasized in the previous section, the renormalization scheme independent corrections to the three point functions do not contain any two body terms and in particular, there are no self energy terms. Therefore there is an apparent puzzle: we show below, the fact that even the corrections to structure constants are determined by the anomalous dimension Hamiltonian is due to important cancellations which take place in the evaluation of (3.4)

3.1 Evaluation of corrections to structure constants

We first evaluate the diagram U^{ij}_{kl} thought of as a 3 body term. Consider 4 scalars, 2 of them with indices i and j being nearest neighbour letters on the operator O_α, As they belong to the same operator they are at the same position. But to regularize the resulting diagrams we use the method of point split regularization, therefore we split them such that the operator with index i is at x_1, while the operator with index j is at x_2 with $x_2 - x_1 = \epsilon$, and $\epsilon \to 0$. Let the index k label the letter of operator O_β at position x_3 and the index l label the letter of operator O_γ at position x_4.

The two process that contribute to $U^{ij}_{kl}(\text{3pt})$ are the quartic interaction of scalars and the interaction due to the intermediate gauge exchange. Therefore

$$U^{ij}_{kl} = Q^{ij}_{kl} + G^{ij}_{kl}, \tag{3.5}$$

where Q^{ij}_{kl} refers to the quartic interaction and G^{ij}_{kl} refers to the gauge exchange diagram. Evaluating each of the diagrams we obtain:

$$Q^{ij}_{kl} = \lim_{x_2 \to x_1} \left(2\delta^j_k\delta^i_l - \delta^i_k\delta^j_l - \delta^{ij}\delta_{kl}\right) \frac{1}{x_{13}^2 x_{24}^2}\phi(r,s), \tag{3.6}$$

here the $SO(6)$ structure arises from the quartic potential of the scalars in $\mathcal{N}=4$ super Yang-Mills, $\phi(r,s)$ is the quartic tree interaction given by

$$\int d^4u \frac{1}{(x_1-u)^2(x_2-u)^2(x_3-u)^2(x_4-u)^2} = \frac{\pi^2\phi(r,s)}{x_{13}^2x_{24}^2}, \tag{3.7}$$

and r and s are the conformal cross ratios given by

$$r = \frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2}, \qquad s = \frac{x_{14}^2x_{23}^2}{x_{13}^2x_{24}^2}. \tag{3.8}$$

Note that as $x_2 \to x_1$, $r \to 0$ and $s \to 1$. Therefore to evaluate the limit in (3.6) we can use the expansion of $\phi(r,s)$ given in (B.5), substituting this expansion in (3.6) we obtain

$$Q_{kl}^{ij} = \left(2\delta_k^j\delta_l^i - \delta_k^i\delta_l^j - \delta^{ij}\delta_{kl}\right)\frac{1}{x_{13}^2x_{14}^2}\left(\ln(\frac{x_{13}^2x_{14}^2}{x_{34}^2\epsilon^2}) + 2\right), \tag{3.9}$$

where we have also kept the log term for completeness. The gauge interaction is given by

$$G_{kl}^{ij} = \lim_{x_2\to x_1} \delta_k^i\delta_l^j H \tag{3.10}$$

where

$$H = (\partial_1-\partial_3)\cdot(\partial_2-\partial_4)\int \frac{d^4ud^4v}{\pi^2(2\pi)^2}\frac{1}{(x_1-u)^2(x_3-u)^2}\frac{1}{(u-v)^2}\frac{1}{(x_2-v)^2(x_3-v)^2}. \tag{3.11}$$

It can be shown that $H(x_1,x_2,x_3,x_4)$ in the above expression can be rewritten entirely in terms of $\phi(r,s)$ by the following identity used in [34]:

$$\begin{aligned} H &= E + C_1 + C_2 + C_3 + C_4, \\ &= (r-s)\frac{1}{x_{13}^2x_{24}^2}\phi(r,s) \\ &+ (s'-r')\frac{\phi(r',s')}{x_{13}^2x_{24}^2} \ \text{ with } r' = \frac{x_{34}^2}{x_{24}^2}, s' = \frac{x_{23}^2}{x_{24}^2};\ 1\to\infty \ \text{ collapse} \\ &+ (s'-r')\frac{\phi(r',s')}{x_{13}^2x_{24}^2} \ \text{ with } r' = \frac{x_{34}^2}{x_{13}^2}, s' = \frac{x_{14}^2}{x_{13}^2};\ 2\to\infty \ \text{ collapse} \\ &+ (s'-r')\frac{\phi(r',s')}{x_{13}^2x_{24}^2} \ \text{ with } r' = \frac{x_{12}^2}{x_{24}^2}, s' = \frac{x_{14}^2}{x_{24}^2};\ 3\to\infty \ \text{ collapse} \\ &+ (s'-r')\frac{\phi(r',s')}{x_{13}^2x_{24}^2} \ \text{ with } r' = \frac{x_{12}^2}{x_{13}^2}, s' = \frac{x_{23}^2}{x_{13}^2};\ 4\to\infty \ \text{ collapse.} \end{aligned} \tag{3.12}$$

E, C_1, C_2, C_3, C_4 are defined respectively by the remaining lines of the above equation. We have labelled r' and s' that occur in the second line of the above equation by

$1 \to \infty$ collapse since these values are obtained by taking the indicated limit in r and s given in (3.8). All other values of r' and s' are obtained using the corresponding limits mentioned above. We will refer to these terms as collapsed diagrams. On substituting (3.12) in the formula for the gauge interaction given in (3.10) we need to take the limit $x_2 \to x_1$. Under this limit $r' \to 0, s' \to 1$ for the C_3 and C_4 collapsed diagrams, but the r' and s' of the remaining C_1 and C_2 collapses do not tend of these values. On examining the expansion of $\phi(r', s')$ given in (B.5) we see that these collapsed diagrams do not reduce to either logarithms or constants under the limit $x_2 \to x_1$, but remain nontrivial functions. Thus the collapses C_1 and C_2 seem to violate conformal invariance, since conformal invariance of the 3 point function predicts that the one loop correction terms must be either logarithms or constants. We will call these collapses dangerous collapses. However in the next subsection we will show that on summing over all the terms given in (3.4), these dangerous collapses cancel leaving behind only logarithms or constants. For the present, let us assume that these collapses cancel and evaluate the remaining terms, they are given by

$$\begin{aligned} G^{ij}_{kl}(\text{3pt}) = \delta^i_k \delta^j_l \Bigg(& -\frac{1}{x_{13}^2 x_{14}^2} \left[\ln\left(\frac{x_{13}^2 x_{14}^2}{x_{34}^2 \epsilon^2}\right) + 2 \right] \\ & + \frac{1}{x_{13}^2 x_{14}^2} \left[\ln\left(\frac{x_{14}^2}{\epsilon^2}\right) + 2 \right] + \frac{1}{x_{13}^2 x_{14}^2} \left[\ln\left(\frac{x_{13}^2}{\epsilon^2}\right) + 2 \right] \Bigg) . \end{aligned} \tag{3.13}$$

The first term in the square bracket is obtained by taking the limit $x_2 \to x_1$ in the first term E of (3.12) and the last two terms are obtained by taking the same limit in the C_3 and C_4 collapsed diagrams of (3.12). Here we have ignored the C_1 and C_2 collapses of of (3.12), as we will show that in the combination in (3.4) they cancel. Combining all the constants to write $U^{ij}_{kl}(\text{3pt})$ we obtain

$$U^{ij}_{kl}(\text{3pt}) = \left[2\left(2\delta^j_k \delta^i_l - \delta^i_k \delta^j_l - \delta^{ij}\delta_{kl}\right) + (-2+2+2)\delta^i_k \delta^j_l \right] . \tag{3.14}$$

In the second term we have written the constant contributions from the first term in (3.13) and the two collapses separately.

We now evaluate $U^{ij}_{kl}(\text{2pt})$: the calculation is similar to the 3 body case, except that we also need to take the limit $x_4 - x_3 = \epsilon$ and $\epsilon \to 0$. This is because in the present calculation the letters ϕ^k and ϕ^l are nearest neighbours on the same operator. Going through the same steps we obtain the following contributions for the quartic

term

$$Q^{ij}_{kl}(2\mathrm{pt}) = \lambda 2\left(2\delta^j_k\delta^i_l - \delta^i_k\delta^j_l - \delta^{ij}\delta_{kl}\right). \tag{3.15}$$

This contribution is identical to the case of the 3 body calculation. For the gauge exchange interaction, all the 4 collapses, including C_1 and C_2, will give rise to logarithms and constants. This is because under the limit $x_4 \to x_3$, the corresponding r' and s' of C_1 and C_2 tends to 0 and 1 respectively. Therefore the constants from the collapses will be twice that of the 3 body calculation. This is is given by

$$G^{ij}_{kl}(2\mathrm{pt}) = (-2+2+2+2+2)\delta^i_k\delta^j_l, \tag{3.16}$$

where we have separated out the contribution of E in (3.12) and the 4 collapses. Thus the sum of quartic interaction and the gauge exchange to the two body terms is given by

$$U^{ij}_{kl}(2\mathrm{pt}) = 2\left(2\delta^j_k\delta^i_l - \delta^i_k\delta^j_l - \delta^{ij}\delta_{kl}\right) + (-2+2+2+2+2)\delta^i_k\delta^j_l. \tag{3.17}$$

With all the ingredients in place, we can evaluate the renormalization scheme independent correction to the structure constant. This is given by

$$\begin{aligned} U^{ij}_{kl}(3\mathrm{pt}) - \frac{1}{2}U^{ij}_{kl}(2\mathrm{pt}) &= \left(2\delta^j_k\delta^i_l - 2\delta^i_k\delta^j_l - \delta^{ij}\delta_{kl}\right), \\ &= \mathcal{H}^{ij}_{kl}, \end{aligned} \tag{3.18}$$

where we have substituted (3.14) and (3.17). Note that since the constant contribution of the collapses in the 2 body diagram are double that of the 3 body, they cancel in the renormalization scheme independent combination. The gauge exchange diagram finally just contributes an additional $-\delta^i_k\delta^j_l$ to give precisely the anomalous dimension Hamiltonian. Substituting (3.18) in (3.4) and summing over all possible planar contractions we will obtain (3.2) which is what we set out to prove.

Let us compare this calculation with the anomalous dimension calculation of [17] and [18]. There one focuses on the terms proportional to the logarithm of the quartic, the gauge exchange and the self energy diagrams. The way the Hamiltonian $\mathcal{H}$ appears is because the self energy contributions cancel all the 4 collapsed diagrams of the gauge exchange leaving behind only the quartic Q and the diagram E, which results in the anomalous dimension Hamiltonian $\mathcal{H}$. As we have seen the appearance of the anomalous dimension calculation in the one loop calculation of the structure constants is entirely due to a different mechanism.

3.2 Cancellation of the dangerous collapsed diagrams

In this subsection we show that the dangerous collapses in (3.12) cancel out when one adds all the three terms in (3.4). The dangerous collapses when two of the indices i_a and i_{a+1} are on the same operator O_α is given by

$$\begin{aligned} D(1;34) = & \lim_{x_2\to x_1} \delta^{i_a}_{j_{a+1}} \delta^{i_{a+1}}_{k_a} \delta^{j_a}_{k_{a+1}} \times \\ & \Bigg((s'-r')\frac{\phi(r',s')}{x_{13}^2 x_{24}^2} \ \text{ with } \ r' = \frac{x_{34}^2}{x_{24}^2}, s' = \frac{x_{23}^2}{x_{24}^2};\ 1\to\infty \ \text{ collapse} \\ & + (s'-r')\frac{\phi(r',s')}{x_{13}^2 x_{24}^2} \ \text{ with } \ r' = \frac{x_{34}^2}{x_{13}^2}, s' = \frac{x_{14}^2}{x_{13}^2};\ 2\to\infty \ \text{ collapse} \Bigg). \end{aligned} \tag{3.19}$$

The dangerous collapse when the indices j_a and j_{a+1} are on the same operator O_β is given by

$$\begin{aligned} D(3;41) = & \lim_{x_2\to x_3} \delta^{i_a}_{j_{a+1}} \delta^{i_{a+1}}_{k_a} \delta^{j_a}_{k_{a+1}} \times \\ & \Bigg((s'-r')\frac{\phi(r',s')}{x_{13}^2 x_{24}^2} \ \text{ with } \ r' = \frac{x_{14}^2}{x_{34}^2}, s' = \frac{x_{13}^2}{x_{34}^2};\ 2\to\infty \ \text{ collapse} \\ & + (s'-r')\frac{\phi(r',s')}{x_{13}^2 x_{24}^2} \ \text{ with } \ r' = \frac{x_{14}^2}{x_{12}^2}, s' = \frac{x_{24}^2}{x_{12}^2};\ 3\to\infty \ \text{ collapse} \Bigg). \end{aligned} \tag{3.20}$$

Note that, here the limit is such $x_2 \to x_3$, this is because two letters are on operator O_β which is at x_3. The index structure is identical to that of previous case in (3.19). Finally, the values of r' and s' is such that the on taking the limit in (3.20) and (3.19), the last line of the (3.20) identically cancels the 1st line of (3.19) when one uses the fact $\phi(r,s)$ is a symmetric function in r and s [4]. Basically the r' and s' of the collapse $2\to\infty$ of (3.19) exchanges with that of the dangerous collapse $3\to\infty$ of (3.20). Let us now write the dangerous collapses when the indices k_a and k_{a+1} are on operator O_γ which is at position x_4.

$$\begin{aligned} D(4;13) = & \lim_{x_2\to x_4} \delta^{i_a}_{j_{a+1}} \delta^{i_{a+1}}_{k_a} \delta^{j_a}_{k_{a+1}} \times \\ & \Bigg((s'-r')\frac{\phi(r',s')}{x_{13}^2 x_{24}^2} \ \text{ with } \ r' = \frac{x_{13}^2}{x_{34}^2}, s' = \frac{x_{14}^2}{x_{34}^2};\ 2\to\infty \ \text{ collapse} \\ & + (s'-r')\frac{\phi(r',s')}{x_{13}^2 x_{24}^2} \ \text{ with } \ r' = \frac{x_{13}^2}{x_{12}^2}, s' = \frac{x_{23}^2}{x_{12}^2};\ 4\to\infty \ \text{ collapse} \Bigg) \end{aligned} \tag{3.21}$$

[4] $\phi(r,s)=\phi(s,r)$ is shown in appendix B.

It is now clear from (3.19), (3.20) and (3.21), that after taking the limits indicated and using the fact $\phi(r,s)$ is a symmetric function in r and s we see that the sum of the dangerous collapses among all the three body terms cancel

$$D(1;34) + D(3;41) + D(4;13) = 0 \tag{3.22}$$

This mechanism of cancellation of dangerous collapses cannot hold when structure constant of interest is of a length conserving process. This is because in a length conserving process the only genuine three body diagrams are when the two nearest neighbour letters are on the longest operator say on O_α and the rest of the letters are on O_β and O_γ. Therefore we cannot possibly have the last two terms in (3.22). But, as we have mentioned in the previous section, in a length conserving process there is a possibility of non-nearest neighbour interactions which are planar. This is shown in fig. 6. If one keeps track of the $U(N)$ group theoretical factors, it is easy to show

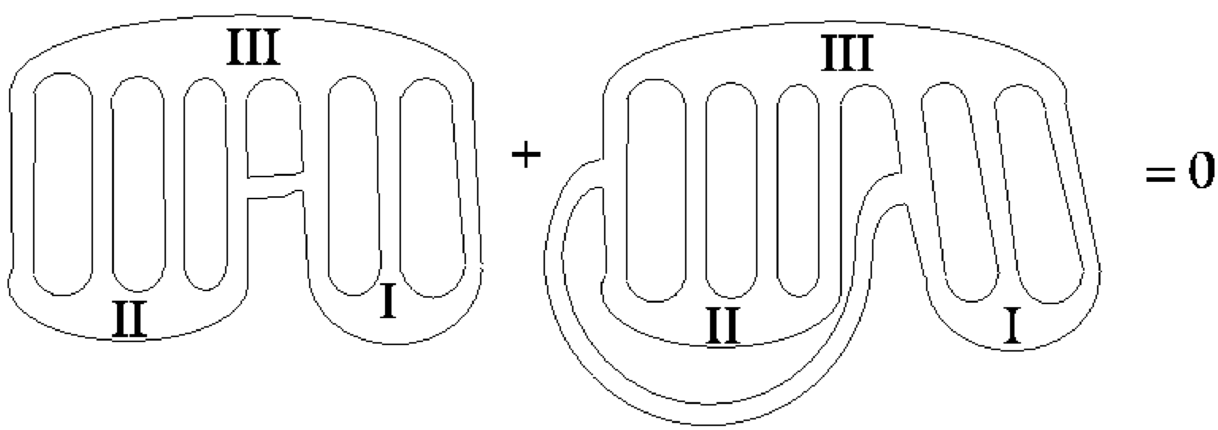

Figure 6: Cancellations in a length conserving process

that there is a relative negative sign between the diagrams in fig. 6. Therefore such diagrams cancel, though we will not go into details in this paper, we have checked that for length conserving process such diagrams ensure that the dangerous collapses in a length conserving process also cancel.

3.3 An example

In this subsection we consider a simple example to illustrate the calculation of one

loop corrections to structure constants. We consider the following operators:

$$O_\alpha = \frac{1}{\sqrt{N^3}}\mathrm{Tr}(\phi^1\phi^2\phi^3), \quad O_\beta = \frac{1}{\sqrt{N^3}}\mathrm{Tr}(\phi^1\phi^2\phi^4), \quad O_\gamma = \frac{1}{N}\mathrm{Tr}(\phi^3\phi^4), \tag{3.23}$$

the operators are at positions x_1, x_3 and x_4 respectively. The tree level correlation function of these operators are given by

$$\langle O_\alpha O_\beta O_\gamma\rangle^{(0)} = \frac{1}{N}\frac{1}{x_{13}^4 x_{14}^2 x_{34}^2}. \tag{3.24}$$

The one loop corrections will all have the above position dependent factor multiplying the λ dependent corrections. Below we write down the corrections from various diagrams, we divide the contributions from genuine three body terms and two body terms. As we have seen in the previous section, we do not have to keep track of the constants from the two body terms as they cancel in the metric subtractions. Therefore we need to look at only the terms proportional to the logarithm in the two body terms. The corrections to the structure constant will be evaluated by (3.2).

Three body terms

The three body terms consist of:

$$\begin{aligned} 2\ &[(Q+E+C_3+C_4)(1;34) + (Q+E+C_3+C_4)(3;41) \\ +\ &(C_3+C_4)(4;13)]\,, \end{aligned} \tag{3.25}$$

here the labels $(1;34)$ refers to the diagram with two letters on the operator O_α and the remaining two letters on the operators O_β and O_γ respectively. We have also suppressed the $SO(6)$ index structure of each diagram for convenience, they can easily be reinstated and evaluated. Note that among the collapsed diagrams we have written down only the contributions of the $3\to\infty$ and $4\to\infty$ collapse since the remaining collapses are dangerous and cancel out. For the diagrams of the type $(4;13)$ we have not written the quartic term Q and E, this is is because on examining the $SO(6)$ structure of these diagrams we see that they cancel among each other. There is an overall factor of 2 because of the presence of the outer three body diagrams. We now give the terms proportional to the logarithm of the above diagrams:

$$2\left(-2\log\left(\frac{x_{13}^2 x_{14}^2}{x_{34}^2\epsilon^2}\right) + \log\left(\frac{x_{13}^2}{\epsilon^2}\right) + \log\left(\frac{x_{14}^2}{\epsilon^2}\right)\right. \tag{3.26}$$

$$
\begin{aligned}
&- 2\log\left(\frac{x_{34}^2 x_{13}^2}{x_{14}^2\epsilon^2}\right) + \log\left(\frac{x_{13}^2}{\epsilon^2}\right) + \log\left(\frac{x_{34}^2}{\epsilon^2}\right) \\
&+ \ \log\left(\frac{x_{14}^2}{\epsilon^2}\right) + \log\left(\frac{x_{34}^2}{\epsilon^2}\right)\bigg) .
\end{aligned}
$$

The logarithms in the above equation are the contributions of the respective terms in (3.25). Using (3.2), the renormalization group invariant correction to the structure constant is given by

$$
\mathcal{H}_{23}^{23} + \mathcal{H}_{24}^{24} + \mathcal{H}_{34}^{34} + \mathcal{H}_{34}^{43} + \mathcal{H}_{13}^{13} + \mathcal{H}_{14}^{14} + \mathcal{H}_{34}^{34} + \mathcal{H}_{13}^{43} = -8. \tag{3.27}
$$

The indices on $\mathcal{H}$ refer to $SO(6)$ indices of the letters involved. Here the extra terms $\mathcal{H}_{34}^{43}$ is because of the fact that the operator O_γ is an operator of two letters whose position can be interchanged.

Two body terms

As mentioned before, for the two body terms we have to focus only on the log terms. The diagrams which contribute to this are:

$$
(Q + E + C_1 + C_2 + C_3 + C_4)(1;3) \, + \, 2S(1;3) + S(1;4) + S(3;4), \tag{3.28}
$$

where the labels $(1;3)$ indicate which two operators the contributions arise from, we have again suppressed the $SO(6)$ indices for convenience. Note that here all the 4 collapses contribute, S refers to the self energy contributions. Evaluating these contributions we obtain

$$
\begin{aligned}
&- 2\log\left(\frac{x_{13}^4}{\epsilon^4}\right) + 4\log\left(\frac{x_{13}^2}{\epsilon^2}\right) \\
&+ -8\log\left(\frac{x_{13}^2}{\epsilon^2}\right) - 4\log\left(\frac{x_{14}^2}{\epsilon^2}\right) - 4\log\left(\frac{x_{34}^2}{\epsilon^2}\right) .
\end{aligned} \tag{3.29}
$$

Combining (3.26), and (3.29) and (3.27) we find that the log correction and the renormalization group invariant one loop correction to the structure constant is given by

$$
\frac{\lambda}{N}\left(-12\log\left(\frac{x_{13}^2}{\epsilon^2}\right) - 8\right) . \tag{3.30}
$$

Here we have reinstated the factor λ/N which occurs in the corrections to the structure constants.

4. Operators with derivatives

In the previous section we showed that the anomalous dimension Hamiltonian controls the corrections to structure constants in the $SO(6)$ sector. There were basically three reasons for this: (i) the $SO(6)$ spin dependent term factorizes out in the calculations, (ii) $\mathcal{N} = 4$ supersymmetry ensures that quartic term and the gauge exchange terms comes with the same coupling constant, (iii) contributions of all collapsed diagrams canceled. As we have argued in the introduction, since $\mathcal{N} = 4$ super Yang-Mills admits a string dual, the structure constants of the theory should be determined basically by the geometric delta function overlap of the dual string theory. One can see that at $\lambda = 0$ and at large N ensures that three point functions of single trace gauge invariant operators can be written as delta function overlap in a string bit theory [16]. Turning on finite λ renders α' of the string theory finite, and induces nearest neighbour interactions between the bits. Thus, the modifications to structure constants must be only due to effects of interaction in the propagation of the bits, the geometric delta function overlap of the string is invariant. The fact that in the $SO(6)$ sector the one loop corrections to the structure constants is dictated by the anomalous dimension Hamiltonian indicates the possibility that it is only the world sheet Hamiltonian in the bit string theory which is necessary to compute corrections to structure constants. To verify this and to identify the precise operator which is responsible for the propagation of the bits we need to compute one loop corrections to structure constants with more general operators outside the $SO(6)$ scalar sector. Among the three simplifications in the $SO(6)$ sector discussed above, the factorization of $SO(6)$ spin dependent term will not be present if there are derivatives in the letters. This motivates the evaluation of one loop corrections to structure constants of operators with derivatives.

4.1 Primaries with derivatives

Before we start the one loop calculation, we need to specify the operators with derivatives which are conformal primaries that we will be dealing with. We work with operators having $SO(6)$ scalars with arbitrary number of derivatives in a fixed

complex direction. For example the following operator

$$\mathrm{Tr}(D_z^{m_1}\phi^{i_1}D_z^{m_2}\phi^{i_2}\cdots\cdot D_z^{m_j}\phi^{i_j}\cdots), \tag{4.1}$$

where $D_z = \partial_z + ig[A_z,\ \cdot\ ]$ [5] is the covariant derivative in a given complex direction $z = x^2 + ix^3$, m_j refers to the number of derivatives on the $j^{\rm th}$ letter. To construct the primaries at tree level we can ignore the commutator term in the covariant derivative. To construct a conformal primary from such operators we need to know the action of the special conformal transformations K_μ on these states. The action of K_μ on a scalar is given by

$$[K_\mu, \phi] = (2x_\mu x\cdot\partial + 2x_\mu - x^2\partial_\mu)\phi. \tag{4.2}$$

Since all the fields are at the origin and the derivatives are only in the holomorphic direction we can set all other coordinates in K_z to zero, this gives

$$K^z = z^2\partial_z + z, \tag{4.3}$$

similarly the other generators are given by

$$P_z = \partial_z, \quad D = 1 + z\partial_z. \tag{4.4}$$

They satisfy the algebra

$$[D, K^z] = K^z, \quad [D, P_z] = -P_z \quad [P_z, K^z] = 2z\partial_z + 1 = D + M_{z\bar{z}} \tag{4.5}$$

where $M_{z\bar{z}} = z\partial_z$ is the angular momentum generator on the z plane when $\bar{z}$ is set to zero. The above algebra forms an $SL(2)$ algebra, to see this identify

$$J_3 = -\frac{1}{2}\left(D + M_{z\bar{z}}\right), \qquad J_+ = P_z, \qquad J_- = K^z, \tag{4.6}$$

then we have

$$[J_3, J_\pm] = \pm J_\pm, \quad [J_+, J_-] = -2J_3. \tag{4.7}$$

Thus scalars with derivatives in a given holomorphic sector form representations of the $SL(2)$ algebra. The action of K_z a scalar with m derivatives is given by

$$[K_z, \frac{\partial^m}{m!}\phi^i] = m\frac{1}{(m-1)!}\partial^{m-1}\phi^i. \tag{4.8}$$

[5] In our notation $g^2 = \frac{g_{YM}^2}{2(2\pi)^2}$, see appendix A.

Here we have divided the mth derivative by $m!$ to ensure the two point function of these derivatives are normalized to 1, we have also suppressed the subscript z on the derivatives which will be understood for the rest of paper. It is easy to construct primaries by suitably taking linear combinations of these operators. For example a simple class of primaries with derivatives only on two of the scalars is given by

$$\sum_{m=0}^{n}(-1)^m\, {}^nC_m \mathrm{Tr}\left(\frac{\partial^m\phi^{i_1}}{m!}\phi^{i_2}\cdots\frac{\partial^{n-m}\phi^{i_j}}{(n-m)!}\phi^{i_{j+1}}\cdots\right). \tag{4.9}$$

Similarly, combinations of operators with derivatives only in the anti-holomorphic direction $\bar{z}$ can be chosen so that they are primaries.

Three point functions as well as two point functions of primaries have definite tensor structure as given in (2.4) and (2.1) respectively. Therefore it is sufficient to focus terms proportional to products of of the identity $\delta_{\mu\nu}$ in the tensor structure. For operators with derivatives only in the holomorphic or the anti-holomorphic direction it is sufficient to look at terms proportional to products of the identity $\delta_{z\bar{z}}$. This simplifies calculations considerably: for instance in the calculation of the interaction with 4 letters, the number of holomorphic derivatives must equal the number of anti-holomorphic derivatives. Finally, another useful fact about the $SL(2)$ sector is that when the scalars are in a given Cartan direction of $SO(6)$, the detailed calculation of the the anomalous dimension Hamiltonian has been done in [19].

4.2 The processes

From the slicing argument and our detailed discussion for the $SO(6)$ sector, the corrections to the structure constants are governed by the constants in the following basic quantity

$$\begin{aligned} &\left(U^{(i_a,m_a)(i_{a+1},m_{a+1})}_{(j_{b+1},n_{b+1})(k_c,s_c)}(3\mathrm{pt})-\frac{1}{2}U^{(i_a,m_a)(i_{a+1},m_{a+1})}_{(j_{b+1},n_{b+1})(k_c,s_c)}(2\mathrm{pt})\right)\delta^{j_b}_{k_{c+1}}\delta(n_b,s_{c+1}) \\ +&\left(U^{(j_b,n_b)(j_{b+1},n_{b+1})}_{(k_{c+1},s_{c+1})(i_a,m_a)}(3\mathrm{pt})-\frac{1}{2}U^{(j_b,n_b)(j_{b+1},n_{b+1})}_{(k_{c+1},s_{c+1})(i_a,m_a)}(2\mathrm{pt})\right)\delta^{k_c}_{i_{a+1}}\delta(s_c,m_{a+1}) \\ +&\left(U^{(k_c,s_c)(k_{c+1},s_{c+1})}_{(i_{a+1},m_{a+1})(j_b,n_b)}(3\mathrm{pt})-\frac{1}{2}U^{(k_c,s_c)(k_{c+1},s_{c+1})}_{(i_{a+1},m_{a+1})(j_b,n_b)}(2\mathrm{pt})\right)\delta^{i_a}_{j_{b+1}}\delta(m_a,n_{b+1}). \end{aligned} \tag{4.10}$$

In the above formula i, j, k label $SO(6)$ indices and m, n, s label the number of derivatives which could be either holomorphic or anti-holomorphic. a, b, c refers to the position of the letters in each of the operators. $\delta(m, n)$ refers to the delta function

which is one when either the number of holomorphic m equals the number of anti-holomorphic derivatives n or vice versa. To further simplify our analysis we will restrict our attention to the cases when the total number of holomorphic derivatives on the operator with 2 letters adjacent to each other in the interaction, is always greater that the number of anti-holomorphic derivatives on either of the letters of the remaining two operators. But, the methods developed here can be applied to study the other cases also. Let us work with only holomorphic derivatives on O_α and anti-holomorphic derivatives on O_β and O_γ. Then, our restriction implies that for the first term in (4.10) $m_a + m_{a+1} \geq n_{b+1}, s_c$.

We now detail all the processes involved in the evaluation of the constants in the interaction $U^{(i,m)(jn)}_{(k,s)(l,t)}$. We again use the point splitting scheme to evaluate the diagrams. For the 3pt contribution the letters $D^m\phi^i/m!$ and $D^n\phi^j/n!$ are at positions x_1 and x_2 respectively such that $x_2 - x_1 = \epsilon$ with $\epsilon \to 0$ and the letters $\bar{D}^s\phi^k/s!$ and $\bar{D}^t\phi^l/t!$ are at x_3 and x_4 respectively. For the 2pt contribution one further takes the limit $x_4 \to x_3 = \epsilon$. In all the diagrams we will first perform the relevant derivatives and then take the appropriate limits. Since we are looking for only the term proportional to the identity we have the constraint $m+n = s+t$, the number of holomorphic derivatives must be equal to the number of anti-holomorphic derivatives.

(i) The quartic interaction

The contribution of the quartic interaction of scalars to $U^{(i,m)(j,n)}_{(k,s)(l,t)}$ is shown in the fig. 7. We first focus on the 3 pt contribution: the constant and the log part of this interaction can be extracted by evaluating the limits in

$$Q^{(i,m),(j,n)}_{(k,s)(l,t)}(3\text{pt}) = \left(2\delta^j_k\delta^i_l - \delta^i_k\delta^j_l - \delta^{ij}\delta_{kl}\right) \lim_{x_2\to x_1} \frac{\partial_1^m\partial_2^n\bar{\partial}_3^s\bar{\partial}_4^t}{m!n!s!t!} \left(\frac{\phi(r,s)}{x_{13}^2 x_{24}^2}\right). \tag{4.11}$$

Now one can use the expansions of $\phi(r,s)$ in (B.5) and perform the appropriate derivatives. In the above equation ∂_1 and ∂_2 refers to the holomorphic derivative in the z_1 and z_2 direction respectively, while $\bar{\partial}_3$ and $\bar{\partial}_4$ refers to the anti-holomorphic derivative in the $\bar{z}_1$ and $\bar{z}_2$ directions respectively. Taking the derivatives is sufficiently simple as one has to focus only on the term proportional to the identity $\delta_{z\bar{z}}$ since we are dealing with primaries, finally one has to take the limit $x_2 \to x_1$. The general

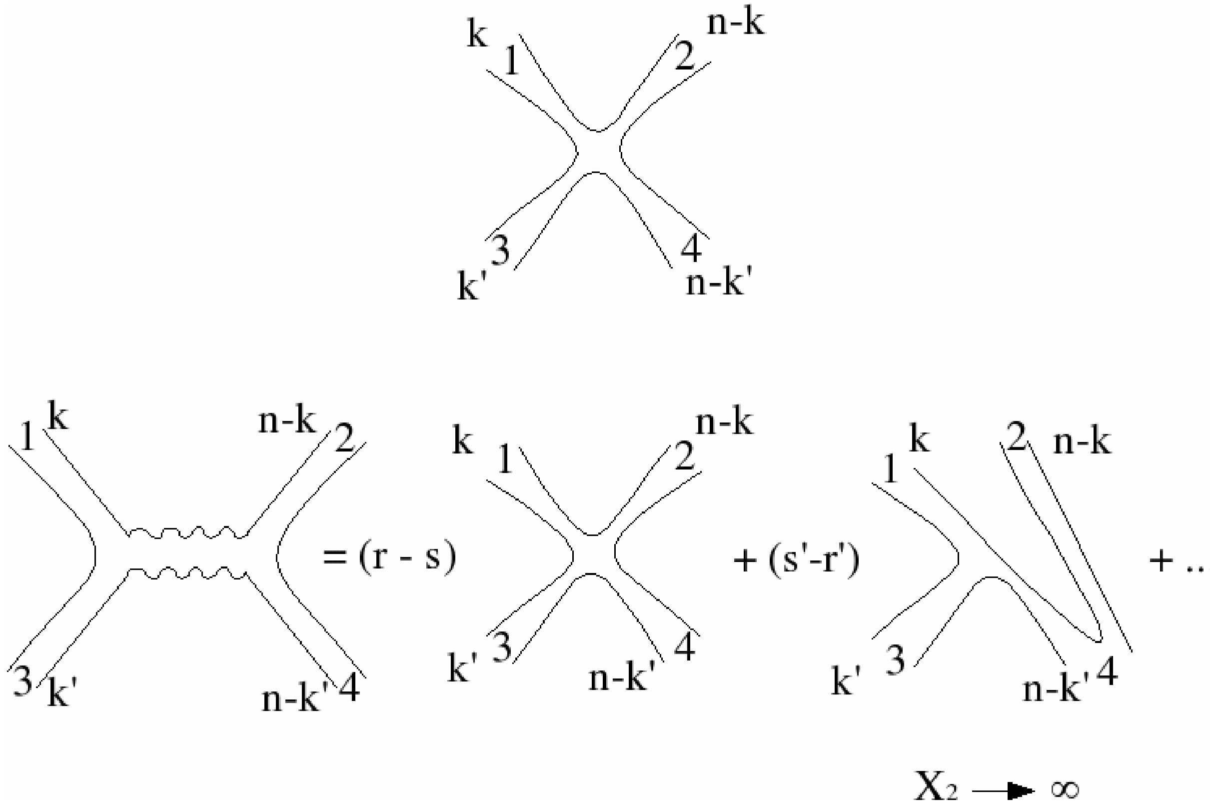

Figure 7: The quartic and the gauge exchange with $x_2 \to \infty$ collapse

form of the quartic term is given by

$$Q^{(i,m),(j,n)}_{(k,s)(l,t)}(3\text{pt}) = \left(2\delta^j_k\delta^i_l - \delta^i_k\delta^j_l - \delta^{ij}\delta_{kl}\right)\frac{1}{x_{13}^{2(s+1)}x_{14}^{2(t+1)}}\left(\mathcal{A}_Q \log\left(\frac{x_{13}^2 x_{14}^2}{x_{34}^2\epsilon^2}\right) + \mathcal{C}_Q\right). \tag{4.12}$$

The coefficient of the log $\mathcal{A}_Q$ and the constant $\mathcal{C}_Q$ for the various cases can be read from table 3. of appendix C. The quartic interaction contribution to the corresponding 2pt term is given by further taking the limit $x_4 \to x_3$, thus the constant obtained for the 2pt term will be the same as constants of the 3pt term.

(ii) Gauge exchange

The gauge exchange contribution to $U(3\text{pt})$ can be found by evaluating the limit in

$$\begin{aligned} G^{(i,m),(j,n)}_{(k,s)(l,t)}(3\text{pt}) &= \delta^i_k\delta^j_l \lim_{x_2\to x_1}\frac{\partial_1^m\partial_2^n\bar{\partial}_3^s\bar{\partial}_4^t}{m!n!s!t!}H, \qquad (4.13)\\ &= \delta^i_k\delta^j_l \lim_{x_2\to x_1}\frac{\partial_1^m\partial_2^n\bar{\partial}_3^s\bar{\partial}_4^t}{m!n!s!t!}\left(E + C_1 + C_2 + C_3 + C_4\right), \end{aligned}$$

where

$$E = (r-s)\frac{\phi(r,s)}{x_{13}^2 x_{24}^2}, \tag{4.14}$$

and C_1, C_2, C_3, C_4 are the collapsed diagrams given in (3.12). In (4.13) we have basically used the (3.12) to write the gauge exchange diagram in terms of the various collapses and (4.14). The equation (3.12) is true when all the points x_1, x_2, x_3, x_4 are strictly distinct. Therefore, we use the equation when all the points are distinct, take the appropriate derivatives and then finally take the limit $x_2 \to x_1$. Just as the quartic diagram, the general form for the diagram $E(\text{3pt})$ is given by

$$E(\text{3pt}) = \delta_k^i \delta_l^j \frac{1}{x_{13}^{2(s+1)} x_{14}^{2(t+1)}} \left(\mathcal{A}_E \log\left(\frac{x_{13}^2 x_{14}^2}{x_{34}^2 \epsilon^2}\right) + \mathcal{C}_E \right). \tag{4.15}$$

In tables 4. and 5 of appendix C. we tabulate the values of $\mathcal{A}_E$ and $\mathcal{C}_E$ for the various cases.

We now examine the structure of the derivatives in each of the collapses and list the conditions under which they contribute to the identity. Consider the $1 \to \infty$ collapse, which is given by

$$C_1 = \delta_k^i \delta_l^j \lim_{x_2 \to x_1} \frac{\partial_1^m \partial_2^n \bar{\partial}_3^s \bar{\partial}_4^t}{m!n!s!t!} \left((r'-s')\frac{\phi(r',s')}{x_{13}^2 x_{24}^2} \right), \tag{4.16}$$
$$\text{with} \qquad r' = \frac{x_{34}^2}{x_{24}^2}, \qquad s' = \frac{x_{23}^2}{x_{24}^2}.$$

Note that if $m > s$ and therefore $n < t$, there is no possibility of saturating the derivatives in the z_1 direction to give a term proportional to the identity, since r' and s' are independent of x_1. Therefore, this collapse diagram contributes to terms proportional to the identity only when $m \le s$ and therefore $n \ge t$. A similar analysis with all the collapses leads to the following table:

Diagram	$m > s;\;\; t > n$	$m < s;\;\; t < n$	$m = s;\;\; n = t$
C_1	No	Yes	Yes
C_2	Yes	No	Yes
C_3	Yes	No	Yes
C_4	No	Yes	Yes

Table 1. Conditions for the contribution of the collapsed diagrams.

It details the conditions on m, n, s, t under which various collapse diagrams contribute to the term proportional to the identity.

Just as in the case of the $SO(6)$ sector discussed in the previous section, the collapses C_1 and C_2 are potentially dangerous as the values of r' and s' for these collapses do not tend to either 0 and 1 respectively under the limit $x_2 \to x_1$. Therefore, C_1 and C_2 are non trivial functions not just logarithms or constants which are required by conformal invariance. As discussed in the previous section for the $SO(6)$ sector, these potentially dangerous collapses must cancel out leaving behind only logarithms or constants. The detailed mechanisms which are responsible for this in this sector will be discussed in the next subsection.

For the evaluation of $G^{(i,m),(j,n)}_{(k,s)(l,t)}(\text{2pt})$ we have to also take $x_4 \to x_3$ limit in addition to the $x_2 \to x_1$ limit. On taking both these limits it is easy to see that r' and s' for the $1 \to \infty$ and $2 \to \infty$ collapse also tend to 0 and 1 respectively. Therefore all the collapses reduce to logs and constants.

(iii) Gauge bosons on one external leg

The covariant derivatives on the letters also have gauge bosons, at one loop one such external gauge boson from say $D^m\phi^i$ can interact with the letters $D^n\phi^j$, $D^t\phi^l$ as show in fig. 8. To evaluate this diagram it is convenient to expand the covariant

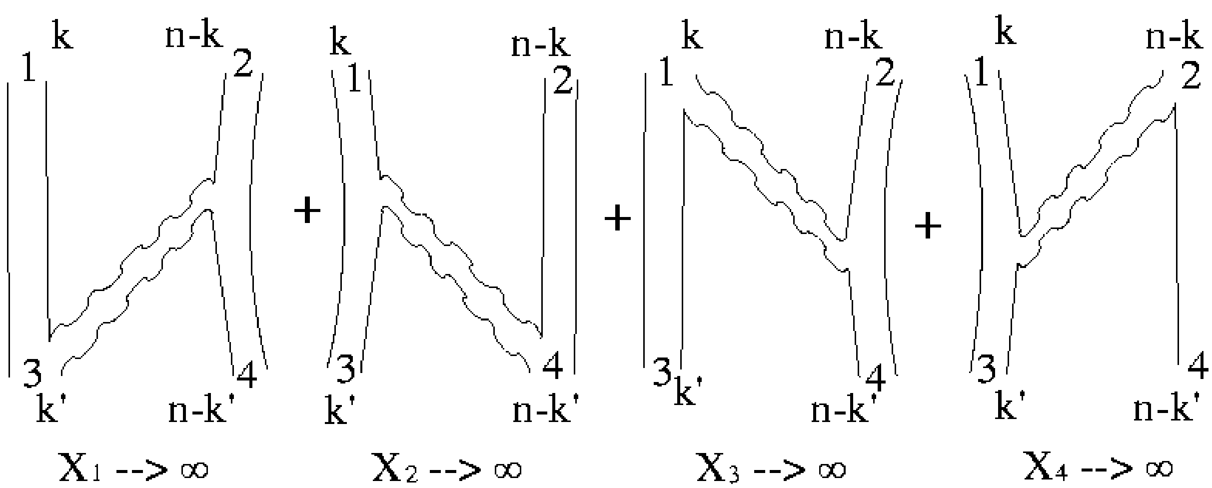

Figure 8: Diagrams with gauge boson on one external leg.

derivative to order one in the g_{YM} as:

$$D^m\phi = \partial^m\phi + ig\sum_{p=1}^{m} {}^mC_p[\partial^{m-1}A_z, \partial^{m-p}\phi]. \tag{4.17}$$

Other similar process with one external gauge boson on the other 3 letters exist, these are shown in fig. 8. We now write the interaction term of each such diagram. The contribution of the diagram with the gauge boson on the letter $D^m\phi^i$ is given by

$$\begin{aligned} A_3(3\mathrm{pt}) = {} & \delta^i_k\delta^j_l\frac{1}{m!n!s!t!}\times \\ & \lim_{x_2\to x_1}\sum_{p=1}^{m} {}^mC_p\left(\partial_1^{m-p}\bar{\partial}_3^s\frac{1}{x_{13}^2}\right)\left(\partial_1^{p-1}(2\partial_2+\partial_1)\partial_2^n\bar{\partial}_4^t\frac{\phi(r',s')}{x_{24}^2}\right), \\ & \text{where} \quad r' = \frac{x_{12}^2}{x_{24}^2},\ s' = \frac{x_{14}^2}{x_{24}^2}. \end{aligned} \tag{4.18}$$

We have labelled this diagram A_3 as the values of r' and s' that occur are the values of the $3 \to \infty$ collapse. Note that we have used momentum conservation on the vertex of a gauge boson with two scalars. From the structure of the derivatives in the first bracket of (4.18), it is clear the term proportional to identity occurs only when $m > s$. Similarly the diagram with the external gauge boson on the letter $D^n\phi^j$ is given by

$$\begin{aligned} A_4(3\mathrm{pt}) = {} & \delta^i_k\delta^j_l\frac{1}{m!n!s!t!}\times \\ & \lim_{x_2\to x_1}\sum_{p=1}^{n} {}^nC_p\left(\partial_2^{n-p}\bar{\partial}_4^t\frac{1}{x_{24}^2}\right)\left(\partial_2^{p-1}(2\partial_1+\partial_2)\partial_1^m\bar{\partial}_3^s\frac{\phi(r',s')}{x_{13}^2}\right), \\ & \text{where} \quad r' = \frac{x_{12}^2}{x_{13}^2},\ s' = \frac{x_{23}^2}{x_{13}^2}. \end{aligned} \tag{4.19}$$

This diagram contributes to terms proportional to the identity only when $n > t$. If the external gauge boson is from the letter $D^s\phi^k$ the interaction is given by

$$\begin{aligned} A_1(3\mathrm{pt}) = {} & \delta^i_k\delta^j_l\frac{1}{m!n!s!t!}\times \\ & \lim_{x_2\to x_1}\sum_{p=1}^{s} {}^sC_p\left(\bar{\partial}_3^{s-p}\partial_1^m\frac{1}{x_{13}^2}\right)\left(\bar{\partial}_3^{p-1}(2\bar{\partial}_4+\bar{\partial}_3)\partial_2^n\bar{\partial}_4^t\frac{\phi(r',s')}{x_{24}^2}\right), \\ & \text{where} \quad r' = \frac{x_{34}^2}{x_{24}^2},\ s' = \frac{x_{23}^2}{x_{24}^2}. \end{aligned} \tag{4.20}$$

Here the above diagram contributes only when $s > m$. Finally when the external gauge boson is from the letter $D^t\phi^l$, the diagram is given by

$$A_2(\text{3pt}) = \delta^i_k\delta^j_l\frac{1}{m!n!s!t!}\times \tag{4.21}$$

$$\lim_{x_2\to x_1}\frac{1}{m!n!s!t!}\sum_{p=1}^{t}{}^tC_p\left(\bar\partial_4^{t-p}\partial_2^n\frac{1}{x_{24}^2}\right)\left(\bar\partial_4^{p-1}(2\bar\partial_3+\bar\partial_4)\partial_1^m\bar\partial_3^s\frac{\phi(r',s')}{x_{13}^2}\right),$$

$$\text{where}\quad r' = \frac{x_{34}^2}{x_{13}^2},\ s' = \frac{x_{14}^2}{x_{13}^2}.$$

This contributes only when $t > n$. We summarize the conditions on m, n, s, t under which all these diagrams contribute to the term proportional to identity in the following table:

Diagram	$m > s;\ \ t < n$	$m < s;\ \ t < n$	$m = s;\ \ n = t$
A_1	No	Yes	No
A_2	Yes	No	No
A_3	Yes	No	No
A_4	No	Yes	No

Table 2. Contributions of diagrams with gauge boson on one leg.

Note that the external gauge boson contribution A_1 and A_2 given in (4.20) and (4.21) respectively are non trivial functions of the respective r' and s', as these do not reduce to either logarithms or constants under the limit $x_2 \to x_1$. Therefore contributions from these diagrams can potentially violate conformal invariance. But, we will show that contributions from these terms add up with the dangerous collapses C_1 and C_2 of (4.13) to finally give only logarithms and constants ensuring conformal invariance. As an indication of this we see that from table 2. and table 1. that whenever A_1 or A_2 contributes to the term proportional to the constant C_1 or C_2 also contributes. The mechanism of how this comes about will be discussed in detail in the next subsection.

(iv) Gauge bosons on two legs

Diagrams with gauge bosons on two different legs contribute constants at one loop. These diagrams are just planar Wick contractions with the gauge bosons on the respective external legs. The ones which contribute to U are the first two diagrams

of fig. 9. The ones with the external gauge boson from the letter $D^m\phi^i$ and $\bar{D}^t\phi^l$ is

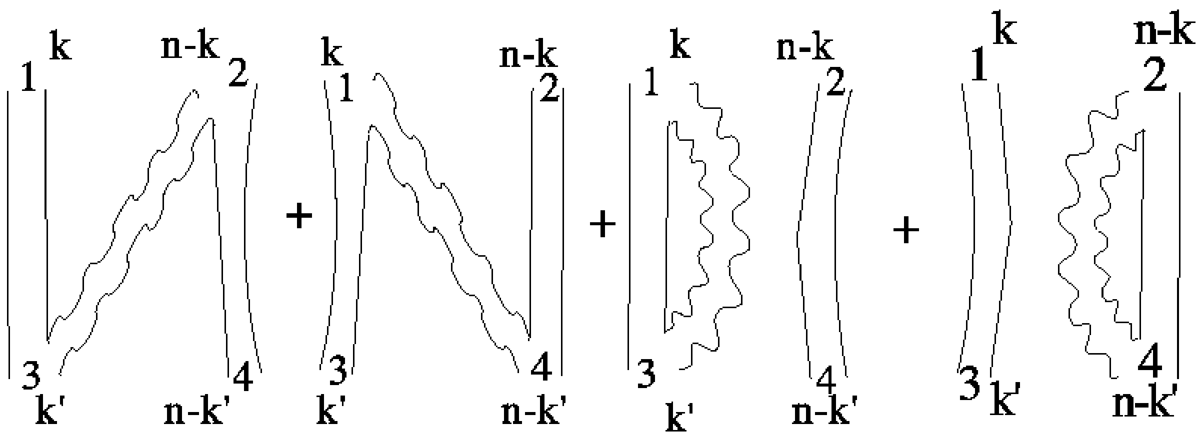

Figure 9: Gauge bosons on two legs

given by

$$B_1 = -2\delta^i_k\delta^j_l \frac{1}{m!n!s!t!} \times \tag{4.22}$$
$$\sum_{p=1}^{m}\sum_{p'=1}^{t} {}^mC_p\, {}^tC_{p'} \bar{\partial}_3^s \partial_1^{m-p}\left(\frac{1}{x_{13}^2}\right)\partial_1^{p-1}\bar{\partial}_4^{p'-1}\left(\frac{1}{x_{14}^2}\right)\partial_2^n\bar{\partial}_4^{t-p'}\frac{1}{x_{24}^2}.$$

The presence of the negative sign in the above formula is due to the fact that the gauge fields on the two legs come on two different sides of the commutator. The factor of 2 occurs in (4.22) if one keeps track the factors of 2 in g^2 and uses the fact that

$$\langle A_z^a(x_1)A_{\bar{z}}^a(x_2)\rangle = \delta^{ab}\frac{1}{2(x_1-x_1)^2}. \tag{4.23}$$

Looking for the term proportional to the identity, we see that the above diagram contributes only when $m > s$ and therefore $n < t$, evaluating the constant we obtain

$$B_1 = -2\delta^i_k\delta^j_l\frac{1}{(m-s)^2}, \tag{4.24}$$

where we have used $m+n = s+t = q$. Similarly the contribution with the external gauge boson from the letter $D^n\phi^j$ and $\bar{D}^s\phi^k$ is given by

$$B_2 = -2\delta^i_k\delta^j_l \frac{1}{m!n!s!t!} \times \tag{4.25}$$
$$\lim_{x_2\to x_1} \sum_{p=1}^{s}\sum_{p'=1}^{n} {}^sC_p\, {}^nC_{p'} \bar{\partial}_3^{s-p}\partial_1^{m}\left(\frac{1}{x_{13}^2}\right)\bar{\partial}_3^{p-1}\partial_2^{p'-1}\left(\frac{1}{x_{23}^2}\right)\partial_2^{n-p'}\bar{\partial}_4^{t}\frac{1}{x_{24}^2}.$$

Again looking for the term proportional to the identity we see that the above term contributes only when $s > m$ and $n > t$. Keeping track of the constant term we see that it is given by

$$B_2 = -2\delta^i_k \delta^j_l \frac{1}{(s-m)^2}. \tag{4.26}$$

Note that both these diagrams do not contribute if $m = s$ or $n = t$.

Consider the remaining contributions from the gauge boson on two legs (see fig. 9.), for instance the diagram with the external gauge boson from the letter $D^m\phi^i$ and $D^s\phi^k$. These diagrams are two body terms and their contribution to the renormalization scheme independent corrections to the three point functions cancel by the slicing argument.

4.3 Mechanisms ensuring conformal invariance

Case 1. $m > s$; $t > n$

From table 1. and table 2. it is clear that only the collapsed diagram C_2 and the external gauge boson on one leg A_2 are the potentially dangerous diagrams which can violate conformal invariance for this case. We show that both these diagrams combine in a non-trivial way to give only logarithms or constants. To simplify matters we first discuss the case of $m = 1, s = 0, n = 0, t = 1$, then C_2 is given by

$$\begin{aligned} C_2 &= \delta^i_k \delta^j_l \partial_1 \bar{\partial}_4 \left(\frac{1}{x_{13}^2 x_{24}^2} (s' - r')\phi(r', s') \right), \qquad r' = \frac{x_{34}^2}{x_{13}^2}, \;\; s' = \frac{x_{14}^2}{x_{13}^2}, \\ &= \delta^i_k \delta^j_l \frac{1}{x_{13}^4 x_{24}^2} \left[-\phi - (s' - r')\partial_{s'}\phi \right], \end{aligned} \tag{4.27}$$

here, in writing the second line we have kept only the terms proportional to the identity while performing the differentiation. The contribution of A_2 can be read out from (4.21), it is given by

$$\begin{aligned} A_2 &= \delta^i_k \delta^j_l \frac{1}{x_{24}^2} \left[(2\bar{\partial}_3 \partial_1 + \bar{\partial}_4 \partial_1) \frac{\phi(r', s')}{x_{13}^2} \right], \\ &= \delta^i_k \delta^j_l \frac{1}{x_{24}^2 x_{13}^4} \left[2\phi + 2(r'\partial_{r'} + s'\partial_{s'})\phi - \partial_{s'}\phi \right]. \end{aligned} \tag{4.28}$$

Adding C_2 and A_2 form (4.27) and (4.28) we obtain

$$C_2 + A_2 = \delta^i_k \delta^j_l \frac{1}{x_{24}^2 x_{13}^4} \left(\phi + (r' + s' - 1)\partial_{s'}\phi + 2r'\partial_{r'}\phi \right). \tag{4.29}$$

Note that on adding C_2 and A_2, the combination of $\phi(r', s')$ in the bracket of the above equation is precisely that of (B.6). In appendix B. it is shown that $\phi(r', s')$ satisfies the inhomogeneous partial differential equation

$$\phi + (r' + s' - 1)\partial_{s'}\phi + 2r'\partial_{r'}\phi = -\frac{\log r'}{s'}. \tag{4.30}$$

The differential equation ensures that though $\phi(r', s')$ is a nontrivial function of r' and s' not just logarithms or constants, the combination which occurs in A_2 and C_2 is such that it reduces to a logarithm ensuring conformal invariance. Substituting this in (4.29) we obtain

$$C_2 + A_2 = \delta_k^i \delta_l^j \frac{1}{x_{24}^2 x_{13}^2 x_{14}^2} \ln\left(\frac{x_{13}^2}{x_{34}^2}\right). \tag{4.31}$$

Now it is also clear that one needs the additional $1/s'$ on the right hand side of(4.30) to obtain the right powers of x dictated by conformal invariance. Finally taking the limit $x_2 \to x_1$ we obtain

$$C_2 + A_2 = \delta_k^i \delta_l^j \frac{1}{x_{14}^4 x_{13}^2} \log\left(\frac{x_{13}^2}{x_{34}^2}\right). \tag{4.32}$$

We have illustrated this mechanism of ensuring conformal invariance in fig. 10

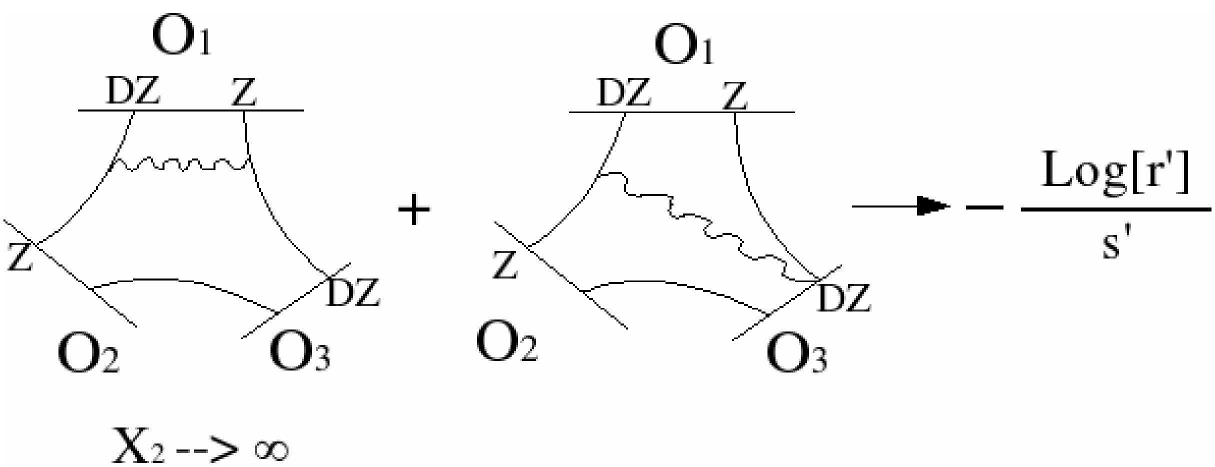

Figure 10: Differential equation ensuring conformal invariance

It is now easy to generalize to the case of arbitrary $m > s; t > n$. For this case, the $2 \to \infty$ collapse is given by

$$C_2 = \delta_k^i \delta_l^j \frac{1}{m!n!s!t!} \partial_1^m \partial_2^n \bar{\partial}_3^s \bar{\partial}_4^t \left(\frac{(s' - r')\phi(r', s')}{x_{13}^2 x_{24}^2}\right), \tag{4.33}$$

$$
\begin{aligned}
&= \delta^i_k \delta^j_l \frac{{}^tC_n}{m!n!s!t!}\left((\partial_2\bar\partial_4)^n \frac{1}{x_{24}^2}\right)\partial_1^m \bar\partial_3^s \bar\partial_4^{t-n-1} \times \\
&\left[\frac{1}{x_{13}^4}(-z_{14}\phi - z_{14}(s'-r')\partial_{s'}\phi)\right].
\end{aligned}
$$

In the second line of the above equation we have first used the Leibnitz rule to move the n derivatives in the direction of $\bar z_4$ to act on the term in the round bracket, then we have focussed only on the term which contributes to the identity $\delta_{z\bar z}$. the term in the square bracket is obtained by the action of one of the remaining $t-n$ $\bar\partial_4$ derivatives on the collapsed term. Now consider A_2, again focusing on the term which contributes to the identity we get

$$
\begin{aligned}
A_2 &= \delta^i_k \delta^j_l \frac{{}^tC_n}{m!n!s!t!}\left((\partial_2\bar\partial_4)^n \frac{1}{x_{24}^2}\right)\partial_1^m \bar\partial_3^s \bar\partial_4^{t-n-1} \times \\
&\left[\frac{1}{x_{13}^4}(2z_{13}\phi + 2z_{13}(r'\partial_{r'} + s'\partial_{s'})\phi - z_{14}\partial_{s'}\phi)\right].
\end{aligned} \tag{4.34}
$$

Here we have only looked at the term $p = t-n$ as it is the only one term in the summation of (4.21) which contributes to the identity. The last line in the above equation is obtained by the action of the operator $(2\bar\partial_3 + \bar\partial_4)$ on $\phi(r',s')/x_{13}^2$. From the structure of derivatives in (4.33) (4.34), it is easy to see that only holomorphic derivatives acting on the term in the square brackets of these equations is ∂_1, Therefore, for the purposes of identifying the term proportional to the identity one can just treat the $z's$ in these brackets as z_1. Then adding (4.33) and (4.34), we see that we can use the differential equation in (4.30) to obtain

$$
C_2 + A_2 = \frac{\delta^i_k\delta^j_l}{m!s!(t-n)!x_{24}^{2(1+n)}}\partial_1^m\bar\partial_3^s\bar\partial_4^{t-n-1}\left[\frac{z_1}{x_{13}^2x_{14}^2}\log\left(\frac{x_{13}^2}{x_{34}^2}\right)\right]. \tag{4.35}
$$

To perform the differentiation in the above equation it is convenient to first do all the $\bar\partial_4$ and the $\bar\partial_3$ derivatives before finally performing the ∂_1 derivatives. This gives

$$
\begin{aligned}
C_2 + A_2 &= \lim_{x_2\to x_1}\frac{\delta^i_k\delta^j_l}{(m-s)x_{24}^{2(1+n)}x_{14}^{2(m-s)}x_{13}^{2(1+s)}}\left(\log\left(\frac{x_{13}^2}{x_{34}^2}\right) + h(s)\right), \\
&= \frac{\delta^i_k\delta^j_l}{(m-s)x_{14}^{2(1+t)}x_{13}^{2(1+s)}}\left(\log\left(\frac{x_{13}^2}{x_{34}^2}\right) + h(s)\right).
\end{aligned} \tag{4.36}
$$

Here we have also written down the final limit to be taken, note that powers of x and the presence of the log or the constant agrees with conformal invariance. Thus,

using the differential equation in (4.30) we have shown that the terms A_2 and C_2 which can potentially violate conformal invariance combine together using (4.30) to restore it. In (4.36) $h(s)$ refers to the harmonic number

$$h(s) = \sum_{j=1}^{s} \frac{1}{s}, \quad s \neq 0, \qquad\qquad h(0) = 0. \tag{4.37}$$

From the tables 1. and 2. we see that the collapse C_3 and the diagram A_3 also contributes when $m > s$. Though these are not dangerous diagrams one can use similar manipulations to sum these. This gives

$$C_3 + A_3 = \frac{\delta_k^i \delta_l^j}{(m-s) x_{14}^{2(1+t)} x_{13}^{2(1+s)}} \left(\log\left(\frac{x_{14}^2}{\epsilon^2}\right) + h(n) \right). \tag{4.38}$$

The total contribution from these graphs is thus obtained by adding (4.36) and (4.38). Note that on adding these terms, the argument of the log is precisely that of what is expected for a three body term.

Case 2. $m < s, \quad t < n$

From table 1. and table 2. we see that the potentially dangerous diagrams are C_1 and A_1. This case is similar to the previous one, going through similar manipulations we can combine these diagrams use (4.30) to give

$$\begin{aligned} C_1 + A_1 &= -\frac{\delta_k^i \delta_l^j \; {}^s C_m}{m! n! s! t!} \left((\partial_1 \bar{\partial}_3)^m \frac{1}{x_{13}^2} \right) \partial_2^n \bar{\partial}_4^t \bar{\partial}_3^{s-m-1} \left(\frac{z_2}{x_{24}^2 x_{23}^2} \log\left(\frac{x_{34}^2}{x_{24}^2} \right) \right), \\ &= \frac{\delta_k^i \delta_l^j}{(s-m) x_{13}^{2(1+m)} x_{24}^{2(1+t)} x_{23}^{2(s-m)}} \left(\log\left(\frac{x_{24}^2}{x_{34}^2} \right) + h(t) \right). \end{aligned} \tag{4.39}$$

Now taking the $x_2 \to x_1$ limit one obtains

$$C_1 + A_1 = \frac{\delta_k^i \delta_l^j}{(s-m) x_{13}^{2(1+s)} x_{14}^{2(1+t)}} \left(\log\left(\frac{x_{14}^2}{x_{34}^2} \right) + h(t) \right). \tag{4.40}$$

Again we see that the terms which can possibly violate conformal invariance add up together to restore conformal invariance. The diagrams C_4 and A_4 for this case can also be combined using similar manipulations to give

$$C_4 + A_4 = \frac{\delta_k^i \delta_l^j}{(s-m) x_{13}^{2(1+s)} x_{14}^{2(1+t)}} \left(\log\left(\frac{x_{13}^2}{\epsilon^2} \right) + h(m) \right). \tag{4.41}$$

Case 3. $m = s, \quad n = t$

From table 1. and table 2. we see that for this case the only diagrams that are potentially dangerous are C_1 and C_2. The mechanisms of how these diagrams are removed is similar to the one for the $SO(6)$ sector discussed in section 2.2. The sum of all the dangerous collapses among the three terms in (4.10) cancel among each other. For notational convenience we choose $m_a = m, m_{a+1} = n, n_{b+1} = s, s_c = t$ in (4.10). Then if the first term has to contribute, we must have $n_b = s_{c+1} = 0$. This is because the operator O_β and O_γ have only anti-holomorphic derivatives and the only way the last free contraction can contribute to the term proportional to the identity is when there are no derivatives present on the corresponding letters. The $SO(6)$ structure of all the three terms involving the dangerous collapses (4.10) is identical so for convenience we suppress them. The dangerous terms from the first term in (4.10) are given by

$$\begin{aligned} D(1;34) = &\lim_{x_2 \to x_1} \frac{1}{(m!)^2 (s!)^2} \frac{1}{x_{34}^2} \times \\ &\Bigg[(\partial_1 \bar{\partial}_3)^m \left(\frac{1}{x_{13}^2} \right) (\partial_2 \bar{\partial}_4)^n \left(\frac{(s'-r')\phi(r',s')}{x_{24}^2} \right) \quad \text{with } r' = \frac{x_{34}^2}{x_{24}^2}, \quad s' = \frac{x_{23}^2}{x_{24}^2} \\ &+ (\partial_2 \bar{\partial}_4)^n \left(\frac{1}{x_{24}^2} \right) (\partial_1 \bar{\partial}_3)^m \left(\frac{(s'-r')\phi(r',s')}{x_{13}^2} \right) \quad \text{with } r' = \frac{x_{34}^2}{x_{13}^2}, \quad s' = \frac{x_{14}^2}{x_{13}^2} \Bigg]. \end{aligned} \tag{4.42}$$

Note that in the above equation we have arranged the derivatives so that it contains the term proportional to the identity. Similarly the dangerous terms from the second term in (4.10) are given by

$$\begin{aligned} D(3;41) = &\lim_{x_2 \to x_3} \frac{1}{(m!)^2 (s!)^2} (\partial_1 \bar{\partial}_4)^n \left(\frac{1}{x_{14}^2} \right) \times \\ &\Bigg[(\partial_1 \bar{\partial}_3)^m \left(\frac{(s'-r')\phi(r',s')}{x_{13}^2 x_{24}^2} \right) \quad \text{with } r' = \frac{x_{14}^2}{x_{13}^2}, \quad s' = \frac{x_{34}^2}{x_{13}^2} \\ &+ (\partial_1 \bar{\partial}_3)^m \left(\frac{1}{x_{13}^2} \right) \left(\frac{(s'-r')\phi(r',s')}{x_{24}^2} \right) \quad \text{with } r' = \frac{x_{14}^2}{x_{24}^2}, \quad s' = \frac{x_{12}^2}{x_{24}^2} \Bigg]. \end{aligned} \tag{4.43}$$

Note that on taking the respective limits we see that the first term of (4.43) cancels the second term of (4.42) as $\phi(r,s)$ is a symmetric function in r and s. Finally the dangerous terms from the last term of (4.10) is given by

$$\begin{aligned} D(4;13) &= C_2 + A_2 \\ &= \lim_{x_2 \to x_4} \frac{1}{(m!)^2 (s!)^2} (\partial_1 \bar{\partial}_3)^m \left(\frac{1}{x_{13}^2} \right) \times \end{aligned} \tag{4.44}$$

$$
\left[(\partial_1\bar{\partial}_4)^n \left(\frac{(s'-r')\phi(r',s')}{x_{23}^2 x_{14}^2}\right) \quad \text{with } r' = \frac{x_{13}^2}{x_{14}^2}, \;\; s' = \frac{x_{34}^2}{x_{14}^2}\right.
$$
$$
\left. + \; (\partial_1\bar{\partial}_4)^n \left(\frac{1}{x_{14}^2}\right)\left(\frac{(s'-r')\phi(r',s')}{x_{24}^2}\right) \quad \text{with } r' = \frac{x_{13}^2}{x_{23}^2}, \;\; s' = \frac{x_{12}^2}{x_{23}^2}\right].
$$

It is now clear that on taking the limits in (4.42), (4.43) and (4.44) the sum vanishes due to pair wise cancellations.

$$
D(1;34) + D(3;41) + D(4;13) = 0. \tag{4.45}
$$

Thus the dangerous collapses completely cancel restoring conformal invariance. We have show this cancellations schematically in the fig. 11

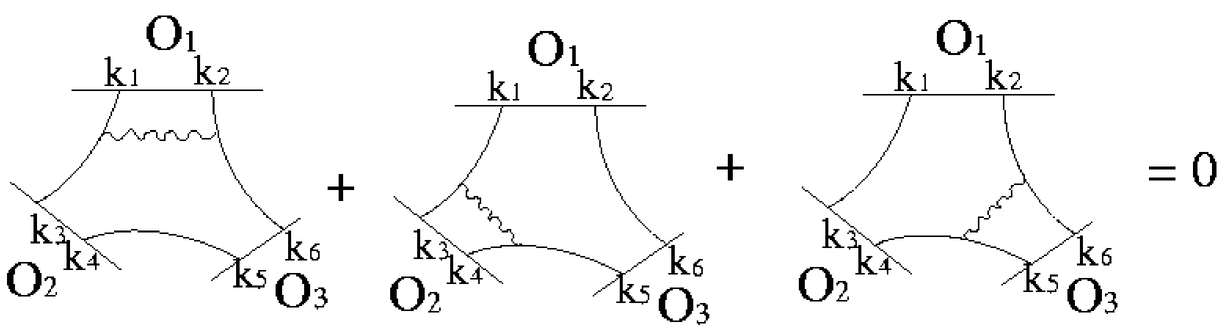

Figure 11: Cancellations among dangerous collapses

From table 1. and table 2. we see that for this case of $m = s$ and $n = t$ the collapse diagrams C_3 and C_4 also contribute. These diagrams are not dangerous. They are given by

$$
C_3 + C_4 = \lim_{x_2 \to x_1} \frac{\delta_k^i \delta_l^j}{(m!)^2 (n!)^2} \times \tag{4.46}
$$
$$
\left[(\partial_1\bar{\partial}_3)^m \left(\frac{1}{x_{13}^2}\right)(\partial_2\bar{\partial}_4)^n \left(\frac{(s'-r')\phi(r',s')}{x_{24}^2}\right) \quad \text{with } r' = \frac{x_{12}^2}{x_{24}^2}, \;\; s' = \frac{x_{14}^2}{x_{24}^2}\right.
$$
$$
\left.(\partial_2\bar{\partial}_4)^n \left(\frac{1}{x_{24}^2}\right)(\partial_1\bar{\partial}_3)^m \left(\frac{(s'-r')\phi(r',s')}{x_{13}^2}\right)\right] \quad \text{with } r' = \frac{x_{12}^2}{x_{13}^2}, \;\; s' = \frac{x_{23}^2}{x_{13}^2}
$$

We can extract the log term and the constant by performing the required differentiations and focusing on the contributions to the identity. For the diagram C_3 and C_4, we do not need to keep track of the constants. The reason is due to a similar phenomenon discussed for the $SO(6)$ sector. To obtain the renormalization group independent constant one needs to subtract the constants from the corresponding two

body term. But, for the two body terms all the collapses C_1, C_2, C_3, C_4 contribute. To find these we just write the diagrams C_1 as in (4.16) and further take the $x_4 \to x_3$ limit. It is then easily seen that the constants from C_1 is identical to the constants from C_3 and the constants from C_2 is identical to the constants from C_4. Therefore in the renormalization group independent contribution

$$C_3(3\text{pt}) + \mathrm{C}_4(3\text{pt}) - \frac{1}{2}\left(\mathrm{C}_1(2\text{pt}) + \mathrm{C}_4(2\text{pt}) + \mathrm{C}_3(2\text{pt}) + \mathrm{C}_4(2\text{pt})\right), \tag{4.47}$$

one finds that the constants cancel. Thus we write just the log terms of (4.46) which contribute to the identity, these are given by

$$C_3 + C_4 = \frac{\delta^i_k \delta^j_l}{x_{13}^{2(m+1)} x_{14}^{2(n+1)}} \left[h(m+1) \log\left(\frac{x_{13}^2}{\epsilon^2}\right) + h(n+1) \log\left(\frac{x_{14}^2}{\epsilon^2}\right)\right]. \tag{4.48}$$

Though we have not emphasized length conserving processes in this paper, we mention that the above mechanism of ensuring conformal invariance for the case of $m = s, n = t$ will not hold for such processes. For a length conserving process, if O_α is the longest operator, then there is only the first term of (4.45), therefore there can be no possibility of cancellation of the dangerous collapses. But, as we have discussed for the case of the $SO(6)$ sector, there are non nearest neighbour interactions which ensure cancellations of the dangerous collapses. This is shown schematically in fig. 12

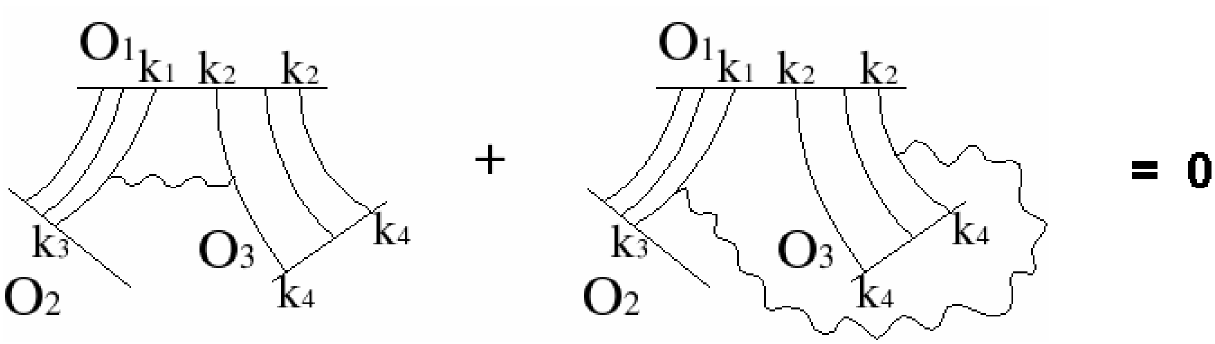

Figure 12: Cancellations in a length conserving process

4.4 Summary of the calculation

Here we summarize the results of our discussion in the previous subsections to give a recipe for the evaluation of one loop corrections to structure constants for the class

of operators with derivatives we are dealing with. We will give the recipe to evaluate the constants in $U(3\text{pt}) - \frac{1}{2}U(2\text{pt})$ for the various cases we have discussed.

(i) Case 1. $m > s, \ \ t > n$

For this case the renormalization group invariant correction to structure constant is given by

$$
\begin{aligned}
U^{(i,m)(j,n)}_{(ks)(lt)}(3\text{pt}) &- \frac{1}{2}U^{(i,m)(j,n)}_{(ks)(lt)}(2\text{pt}) \qquad (4.49)\\
&= \frac{1}{2}\left(V^{ij}_{kl}\mathcal{C}_Q + \delta^i_k\delta^j_l\left(\mathcal{C}_E + C_2 + A_2 + C_3 + A_3 + B_1\right)\right),\\
&= \frac{1}{2}\frac{\lambda}{N}\left(V^{ij}_{kl}\mathcal{C}_Q + \delta^i_k\delta^j_l\left(\mathcal{C}_E + \frac{h(s)}{m-s} + \frac{h(n)}{m-s} - \frac{2}{(m-s)^2}\right)\right).
\end{aligned}
$$

Here $\mathcal{C}_Q$ refers to the constant from the quartic diagram, which can be read out from table 3. of appendix C. $\mathcal{C}_E$ refers to the constant from the diagram E, this can be read out from the tables 4. and 5. V^{ij}_{kl} stands for the $SO(6)$ structure of the quartic given by

$$
V^{ij}_{kl} = 2\delta^j_k\delta^i_l - \delta^i_k\delta^j_l - \delta^{ij}\delta_{kl} \qquad (4.50)
$$

In the last line of (4.49) we have substituted the values constants of the diagrams $C_2 + A_2$, $C_3 + A_3$ and B_1 from (4.35), (4.38) and (4.24) respectively. We have also reinstated the t'Hooft coupling and the $1/N$ factor of the normalization of the structure constant.

(ii) Case 1. $m < s, \ \ t < n$

The renormalization group invariant correction to the structure constant is given by

$$
\begin{aligned}
U^{(i,m)(j,n)}_{(ks)(lt)}(3\text{pt}) &- \frac{1}{2}U^{(i,m)(j,n)}_{(ks)(lt)}(2\text{pt}) \qquad (4.51)\\
&= \frac{1}{2}\left(V^{ij}_{kl}\mathcal{C}_Q + \delta^i_k\delta^j_l\left(\mathcal{C}_E + C_1 + A_1 + C_4 + A_4 + B_2\right)\right),\\
&= \frac{1}{2}\frac{\lambda}{N}\left(V^{ij}_{kl}\mathcal{C}_Q + \delta^i_k\delta^j_l\left(\mathcal{C}_E + \frac{h(t)}{s-m} + \frac{h(m)}{s-m} - \frac{2}{(m-s)^2}\right)\right).
\end{aligned}
$$

Here we have substituted the values of $C_1 + A_1$, $C_4 + A_4$ and B_2 from (4.40), (4.41) and (4.26). The rest of the constants can be read out from the tables in appendix C.

(iii) Case 2. $m = s, \ \ t = n$

As we have discussed earlier for this case the constants from all the collapses cancel in the renormalization group invariant combination given in (4.47). There are no contributions from gauge bosons on two external legs, thus we are left with constants only from the quartic Q and the diagram E, therefore we have

$$\begin{aligned} U^{(i,m)(j,n)}_{(ks)(lt)}(3\mathrm{pt}) - \frac{1}{2}U^{(i,m)(j,n)}_{(ks)(lt)}(2\mathrm{pt}) \\ = \frac{1}{2}\frac{\lambda}{N}\left(V^{ij}_{kl}\mathcal{C}_Q + \delta^i_k\delta^j_l\left(\mathcal{C}_E\right)\right). \end{aligned} \tag{4.52}$$

Again the constants occurring above can be read out from appendix C. As a simple check note that when the number of derivatives are set to zero, evaluating $\mathcal{C}_Q$ and $\mathcal{C}_E$ in the above we obtain the anomalous dimension Hamiltonian $\mathcal{H}$ which determines the corrections to structure constants in the $SO(6)$ sector.

4.5 An example

To illustrate the methods developed we compute the one loop corrections for a simple example of three point function. Consider the following three operators:

$$\begin{aligned} O_\alpha &= \frac{1}{\sqrt{N^3}}\sum_{k=0}^{n} {}^nC_k(-1)^k\mathrm{Tr}(\partial^{n-k}\phi^1\partial^k\phi^2\phi^3), \\ O_\beta &= \frac{1}{\sqrt{N^3}}\sum_{k=0}^{n} {}^nC_k(-1)^k\mathrm{Tr}(\bar\partial^{n-k}\phi^1\bar\partial^k\phi^2\phi^4), \\ O_\gamma &= \frac{1}{N}\mathrm{Tr}(\phi^3\phi^4). \end{aligned} \tag{4.53}$$

where O_α is at position x_1, O_β at x_3 and O_γ at x_4. The tree level correlation function of these operators is given by

$$\langle O_\alpha O_\beta O_\gamma\rangle^{(0)} = \frac{1}{N}\sum_{k=0}^{n}\frac{(\,{}^nC_k)^2}{x_{13}^{2(n+1)}x_{14}^2x_{34}^2} \tag{4.54}$$

Now we compute the one loop corrections to this structure constant. All the corrections, the log terms as well as the renormalization group invariant correction will multiply the position dependent prefactor

$$\frac{1}{x_{13}^{2(n+1)}x_{14}^2x_{34}^2}, \tag{4.55}$$

which is determined by the tree level dimensions of the three operators in (4.53). We write below the log corrections and the renormalization group invariant correction to the structure constant arising from the various diagrams.

Three body terms

The three body interactions consists of the following diagrams:

$$
\begin{aligned}
&2\sum_{k=0}^{n}({}^{n}C_k)^2 \left(Q^{k0}_{k0} + E^{k0}_{k0} + (C_3+C_4)^{k0}_{k0}(1;34)\right. \\
&\left. + \; Q^{k0}_{k0} + E^{k0}_{k0} + (C_3+C_4)^{k0}_{k0}(3;41) + (C_3+C_4)^{00}_{00}(4;13)\right).
\end{aligned} \tag{4.56}
$$

Here we have suppressed the $SO(6)$ indices but kept the indices which indicate the number of derivatives on the letters involved. There are no contributions of $(Q+E)(4;13)$ as the $SO(6)$ structure of these diagrams ensures that they cancel each other. Evaluating the log terms of these diagrams using the tables in appendix C. we find:

$$
\begin{aligned}
&2\sum_{k=0}^{n} {}^{n}C_k)^2 \left(\left[-\frac{2}{k+1} - h(k)\right]\log\left(\frac{x_{13}^2 x_{14}^2}{x_{34}^2\epsilon^2}\right) + h(k+1)\log\left(\frac{x_{13}^2}{\epsilon^2}\right) + \log\left(\frac{x_{14}^2}{\epsilon^2}\right)\right. \\
&+ \left[-\frac{2}{k+1} - h(k)\right]\log\left(\frac{x_{13}^2 x_{34}^2}{x_{14}^2\epsilon^2}\right) + h(k+1)\log\left(\frac{x_{13}^2}{\epsilon^2}\right) + \log\left(\frac{x_{34}^2}{\epsilon^2}\right) \\
&\left. + \; \log\left(\frac{x_{14}^2}{\epsilon^2}\right) + \log\left(\frac{x_{34}^2}{\epsilon^2}\right)\right).
\end{aligned} \tag{4.57}
$$

We have written down each contribution in (4.57), so that they appear in the order of the diagrams in (4.56). To write the renormalization group invariant correction to the structure constants we need to find the constant in each of the terms in (4.56) and perform the metric subtractions. We have already shown that the constants form all the collapses in (4.56) cancel. Therefore we have to look for constants of only the Q's and E's which are listed in appendix C. The metric contributions to these are identical and since they are weighted by 1/2, the final result is just half of the corresponding values listed in appendix C. Writing down these for each of the terms in (4.56) we get

$$
\mathcal{K} = -4\sum_{k=0}^{n}({}^{n}C_k)^2 \times \left(\sum_{l=0}^{k}(-1)^l \, {}^{k}C_l \frac{l+k+2}{(l+1)^2} h(l+1)\right). \tag{4.58}
$$

Note that if the number of derivatives n is set to zero in the above expression we obtain -8 which agrees with (3.27).

Two body terms

As we have discussed before, because of the slicing argument one needs to evaluate only the terms proportional to the logarithm in the two body diagrams. The diagrams are given by

$$\begin{aligned} &\textstyle\sum_{k,k'=0}^{n}\ {}^{n}C_k\ {}^{n}C'_k(-1)^{k+k'}\,(Q+E \\ &+C_1+C_2+C_3+C_4+A_1+A_2+A_3+A_4)^{kn-k}_{k'n-k'}\,(1;3) \\ &+\textstyle\sum_{k=0}^{n}(\ {}^{n}C_k)^2\,(S_k(1;3)+S_{n-k}(1;3)+S_0(1;4)+S_0(3;4))\,, \end{aligned} \tag{4.59}$$

where S_k refers to the self energy contribution of a scalar with k derivatives. The contribution of these self energy diagrams can be read out from [19]. Evaluating the terms proportional to the logarithm of these diagrams we obtain

$$\begin{aligned} &\sum_{k=0}^{n}(\ {}^{n}C_k)^2\left((-2h(k)-\frac{2}{n+1})\log\left(\frac{x_{13}^4}{\epsilon^4}\right)+4h(k+1)\log\left(\frac{x_{13}^2}{\epsilon^2}\right)\right) \\ &+\sum_{k,k',k\neq k'}^{n}\ {}^{n}C_k\ {}^{n}C_{k'}(-1)^{k+k'}\left((\frac{1}{|k-k'|}-\frac{2}{n+1})\log\left(\frac{x_{13}^4}{\epsilon^4}\right)+\frac{2}{|k-k'|}\log\left(\frac{x_{13}^2}{\epsilon^2}\right)\right) \\ &-4\sum_{k=0}^{n}(\ {}^{n}C_k)^2\left[(h(k)+h(k+1)+1)\log\left(\frac{x_{13}^2}{\epsilon^2}\right)+\log\left(\frac{x_{14}^2}{\epsilon^2}\right)+\log\left(\frac{x_{34}^2}{\epsilon^2}\right).\right] \end{aligned} \tag{4.60}$$

Adding the log terms in (4.57) and (4.60) we obtain only terms with $\log(x_{13}^2/\epsilon^2)$. The rest of the log terms cancel, this coefficient is given by:

$$\begin{aligned} &-4\sum_{k=0}^{n}(\ {}^{n}C_k)^2\left(\frac{1}{k+1}+2h(k)+1\right)-4\delta_{n,0} \\ &+\sum_{k,k',k\neq k'}^{n}\ {}^{n}C_k\ {}^{n}C_{k'}(-1)^{k+k'}\left(\frac{4}{|k-k'|}\right). \end{aligned} \tag{4.61}$$

As a simple check, note that on setting $n=0$ the above expression reduces to -12 which was obtained in (3.30).

5. Conclusions

We have evaluated one loop corrections to the structure constants in planar $\mathcal{N}=4$ Yang-Mills for two classes of operators, the $SO(6)$ sector and for operators with derivatives in one holomorphic direction. The summary of the results which enables one to evaluate these structure constants for any operator in these sectors are given

in section 4.4. For the $SO(6)$ scalar sector we find that the one loop anomalous dimension Hamiltonian determines the corrections to the structure constants. The reasons for this are: $\mathcal{N} = 4$ supersymmetry which relates the quartic coupling of scalars to the gauge coupling, the $SO(6)$ spin dependent term factorizes in the calculations and contributions of all the collapsed diagrams canceled. For the sector with derivatives we noticed that essentially the structure constants are determined by a suitable combination of derivatives acting on the fundamental tree function $\phi(r,s)$. Conformal invariance in the calculation was ensured by a linear inhomogeneous partial differential equation satisfied by $\phi(r,s)$ which enabled us to combine the diagrams violating conformal invariance to restore it. The methods developed in this paper can be generalized to the all classes of operators in $\mathcal{N} = 4$ Yang-Mills.

The fact that in the $SO(6)$ sector the one loop corrections to the structure constants are determined by the one loop anomalous dimension Hamiltonian indicates the possibility that in a string bit theory the one loop corrected structure constants can be determined by the delta function overlap with modification in the propagation of the bits taken into account. The immediate suggestion would be that it is the anomalous dimension Hamiltonian which determines the propagation of the bits. In [16] we address this question in detail.

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A. Notations

The action of $\mathcal{N} = 4$ supersymmetric Yang-Mills is best thought of as dimensional reduced maximal supersymmetric Yang-Mills from 10 dimensions. The action is

given by

$$S = \frac{1}{(2\pi)^2}\int d^4x \mathrm{Tr}\left(\frac{1}{4}F_{\mu\nu}^{\mu\nu} + \frac{1}{2}D_\mu\phi^i D^\mu\phi^i - \frac{g^2}{4}[\phi^i,\phi^j][\phi^i,\phi^j] \right. \tag{A.1}$$
$$\left. +\frac{1}{2}\bar{\psi}\Gamma_\mu D^\mu\psi - g\frac{i}{2}\bar{\psi}\Gamma_i[\phi^i,\psi]\right),$$

where A_μ with $\mu = 1,\dots,4$ is the gauge field in 4 dimensions, ψ is a 16 component Majorana-Weyl spinor obtained from the Majorana-Weyl spinor in 10 dimensions. ϕ^i, $i = 1,\dots 6$ are scalars which transform as a vector under the R-symmetry group $SO(6)$. (Γ_μ, Γ_i) are the ten-dimensional Dirac matrices in the Majorana-Weyl representation. All the fields transform in the adjoint representation of the gauge group $U(N)$, to be specific they are $N\times N$ matrices which can be expanded in terms of the generators T^a of the gauge group as

$$\phi^i = \sum_{a=0}^{N^2-1}\phi^{i(a)}T^a, \quad A_\mu = \sum_{a=0}^{N^2-1}A_\mu^{(a)}T^a, \quad \psi = \sum_{a=0}^{N^2-1}\psi^{(a)}T^a. \tag{A.2}$$

The generators T^a satisfy

$$\mathrm{Tr}(T^aT^b) = \delta^{ab}, \qquad \sum_{a=0}^{N^2-1}(T^a)^\alpha_\beta(T^a)^\gamma_\delta = \delta^\alpha_\delta\delta^\gamma_\beta. \tag{A.3}$$

In (A.1) $g^2 = g_{YM}^2/2(2\pi)^2$, [6] the covariant derivatives are given by $D_\mu = \partial_\mu + ig[A_\mu, \cdot\,]$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig^2[A_\mu, A_\nu]$. All our calculations are done in the Feynman gauge. Using the normalization of the action given in (A.1), the tree level two point functions of the scalar and the vector are given by

$$\langle\phi^{i(a)}(x_1)\phi^{j(b)}(x_2)\rangle = \frac{\delta^{ij}\delta^{ab}}{(x_1-x_2)^2}, \tag{A.4}$$
$$\langle A_\mu^{(a)}(x_1)A_\nu^{(b)}(x_2)\rangle = \frac{\delta_{\mu\nu}\delta^{ab}}{(x_1-x_2)^2}.$$

B. Properties of the fundamental tree function

In this appendix we will prove various properties of the fundamental tree function $\phi(r,s)$ defined in (3.7) which are used at various instances in the paper. To obtain a

[6] Our convention differs from [34] in that we have scaled the fields by $g_{YM}/2\pi\sqrt{2}$

series expansion of $\phi(r,s)$ and to show that it satisfies the partial differential equation (4.30) we will use is its integral representation shown in [35]

$$\phi(r,s) = \int_0^1 \frac{-\log(r/s) - 2\log\xi}{s - \xi(r+s-1) + \xi^2 r}\, d\xi. \tag{B.1}$$

From this integral representation we can find a series expansion of $\phi(r,s)$ around $r=0, s=1$, by expanding the denominator in (B.1) as

$$\frac{1}{s-\xi(r+s-1)+\xi^2 r} = \sum_{k,l=0}^{\infty} (-1)^{k+l} \xi^k (\xi-1)^{k+l} \frac{(k+l)!}{k!\ l!} r^k (1-s)^l. \tag{B.2}$$

To perform the series expansion we need the following integrals

$$\begin{aligned} \int_0^1 \xi^k (\xi-1)^{k+l}\ d\xi &= (-1)^{k+l} \frac{k!(k+l)!}{(2k+l+1)!}, \\ \int_0^1 \xi^k(\xi-1)^{k+l} \log\xi\ d\xi &= (-1)^{k+l} \frac{k!(k+l)!}{(2k+l+1)!} \left(h(k) - h(2k+l+1)\right), \end{aligned} \tag{B.3}$$

where $h(n)$ is the harmonic number defined in (4.37). Substituting (B.3) and (B.2) in (B.1) we obtain

$$\begin{aligned} \phi(r,s) &= -\sum_{k,l=0}^{\infty} \frac{(k+l)!^2}{l!(2k+l+1)!} r^k (1-s)^l \log(r/s) \\ &\quad + 2\sum_{k,l=0}^{\infty} \frac{(k+l)!^2}{l!(2k+l+1)!} \left(h(2k+l+1) - h(k)\right) r^k (1-s)^l. \end{aligned} \tag{B.4}$$

Through out the paper we need the expansion of $\phi(r,s)$ at $r=0$, this is given by

$$\begin{aligned} \phi(0,s) &= -\sum_{l=0}^{\infty} \frac{1}{l+1}(1-s)^l \ln(\frac{r}{s}) + 2\sum_{l=0}^{\infty} h(l+1)\frac{1}{l+1}(1-s)^l, \\ &= -\sum_{l=0}^{\infty} \frac{1}{l+1}(1-s)^l \ln(r) + 2\frac{(1-s)^l}{(l+1)^2} \end{aligned} \tag{B.5}$$

Now we show that $\phi(r,s)$ satisfies the following inhomogeneous linear partial differential equations which ensures conformal invariance in the three point function calculations of the paper.

$$\phi(r,s) + (s+r-1)\partial_s \phi(r,s) + 2r\partial_r \phi(r,s) = -\frac{\log r}{s}, \tag{B.6}$$

$$\phi(r,s) + (s+r-1)\partial_r \phi(r,s) + 2s\partial_s \phi(r,s) = -\frac{\log s}{r}. \tag{B.7}$$

To, simplify matters, we introduce the notation

$$D(r,s,\xi) = s - \xi(r+s-1) + \xi^2 r, \tag{B.8}$$

then substituting the integral representation (B.1) of $\phi(r,s)$ in the first equation of (B.6) we obtain

$$\begin{aligned}
&(1+(s+r-1)\partial_s + 2r\partial_r)\phi(r,s) = \\
&\int_0^1 d\xi \frac{1}{D(r,s,\xi)}\left(-\log r/s - 2\log\xi + (s+r-1)/s - 2\right) \\
+ &\int_0^1 d\xi \frac{\log r/s + 2\log\xi}{(D(r,s,\xi))^2}((s+r-1)\partial_s D(r,s,\xi) + 2r\partial_r D(r,s,\xi)).
\end{aligned} \tag{B.9}$$

We can integrate the expression on the second line of the above equation by parts by using the following identity

$$(s+r-1)\partial_s D(r,s,\xi) + 2r\partial_r D(r,s,\xi) = -(1-\xi)\partial_\xi D(r,s,\xi). \tag{B.10}$$

which results in

$$\begin{aligned}
(1+(s+r-1)\partial_s + 2r\partial_r)\phi(r,s) = &\left.\frac{(1-\xi)(\log r/s + 2\log\xi))}{D(r,s,\xi)}\right|_\epsilon^1 + \\
&+ \int_\epsilon^1 d\xi \frac{(s+r-1)/s - 2/\xi)}{D(r,s,\xi)}
\end{aligned} \tag{B.11}$$

Note that we have introduced and parameter ϵ since $\log\xi$ in the first term is divergent at the lower limit. Similarly there is a log divergence in the second term of the above equation. We now show that these divergences cancel each other. Let us write the term contributing to the divergence in the second term of (B.11) as

$$\int_\epsilon^1 d\xi \frac{-2/\xi}{D(r,s,\xi)} = \int_\epsilon^1 d\xi \frac{-2/s}{\xi} + \int_\epsilon^1 d\xi \frac{-2(r+s-1-r\xi)/s}{D(r,s,\xi)} \tag{B.12}$$

Substituting this in (B.11) we obtain

$$\begin{aligned}
(1+(s+r-1)\partial_s + 2r\partial_r)\phi(r,s) = &\left.\frac{\log r/s - \xi(\log r/s + 2\log\xi)}{D(r,s,\xi)}\right|_0^1 \\
&+ \int_0^1 \frac{(-(r+s-1) + 2r\xi)/s}{D(r,s,\xi)}, \\
&= -\frac{\log r/s}{s} + \left.\frac{\log D(r,s,\xi)}{s}\right|_0^1, \\
&= -\frac{\log r}{s}.
\end{aligned} \tag{B.13}$$

Using similar manipulations one can show that $\phi(r, s)$ also satisfies the second partial differential equation in (B.6).

We also use the fact that $\phi(r, s)$ is a symmetric function in r and s. This is best shown using the defining expression of $\phi(r, s)$

$$\phi(r,s) = \frac{x_{13}^2 x_{24}^2}{\pi^2} \int d^4u \frac{1}{(x_1-u)^2(x_2-u)^2(x_3-u)^2(x_4-u)^2}, \tag{B.14}$$

where r and s are given by

$$r = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \qquad s = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}. \tag{B.15}$$

From the definition of r and s above we see that interchange of x_1 and x_3 brings about an interchange of r and s. But the definition (B.14) is easily seen to be invariant under x_1 to x_3. Therefore, we conclude $\phi(r, s)$ is a symmetric function of r and s. $\phi(r, s)$ also satisfies the property

$$\phi(r,s) = \frac{1}{r}\phi(1/r, s/r). \tag{B.16}$$

This can be shown from the fact $r \leftrightarrow 1/r$ and $s \leftrightarrow s/r$ when $x_2 \leftrightarrow x_3$. Then it is easy to see that the symmetry (B.16) is manifest in (B.14). Though these symmetry properties of $\phi(r, s)$ are not manifest in its integral representation given in (B.1), we have seen that through a series of manipulations it is possible to derive these symmetry properties from (B.1).

C. Tables

In the table below we given the values of the coefficient of the logarithm $\mathcal{A}_Q$ and the constant $\mathcal{C}_Q$ of the quartic Q in (4.12).

m	n	s	t	$\mathcal{A}$	$\mathcal{C}$
m	0	m	0	$\frac{1}{m+1}$	$\sum_{l=0}^{m} \frac{2h(l+1)}{l+1}(-1)^l \, {}^mC_l$
m	0	0	m	$\frac{1}{m+1}$	$\frac{2}{(m+1)^2}$
0	m	m	0	$\frac{1}{m+1}$	$\frac{2}{(m+1)^2}$
m	n	s	0	$\frac{1}{s+1}$	$-\frac{h(s)}{s+1} + {}^sC_m \sum_{l=0}^{m}(-1)^{m-l} \, {}^mC_l \left(\frac{h(s-l)}{s-l+1} + \frac{2}{(s-l+1)^2}\right)$
m	n	0	t	$\frac{1}{t+1}$	$-\frac{h(t)}{t+1} + {}^tC_n \sum_{l=0}^{n}(-1)^{n-l} \, {}^nC_l \left(\frac{h(t-l)}{t-l+1} + \frac{2}{(t-l+1)^2}\right)$
m	0	s	t	$\frac{1}{m+1}$	$-\frac{h(m)}{m+1} + {}^mC_s \sum_{l=0}^{s}(-1)^{s-l} \, {}^sC_l \left(\frac{h(m-l)}{m-l+1} + \frac{2}{(m-l+1)^2}\right)$
0	n	s	t	$\frac{1}{m+1}$	$-\frac{h(n)}{n+1} + {}^nC_t \sum_{l=0}^{t}(-1)^{t-l} \, {}^tC_l \left(\frac{h(n-l)}{n-l+1} + \frac{2}{(n-l+1)^2}\right)$

Table 3: $\mathcal{A}_Q$ and $\mathcal{C}_Q$ for the quartic Q.

Note that we have not given the values of $\mathcal{A}_Q$ and $\mathcal{C}_Q$ for the most general case of m, n, s, t. The value of the term proportional to the logarithm $\mathcal{A}_Q$, is always $1/(m + n + 1)$ for arbitrary values of m, n, s, t. The manipulations to extract the constant from (4.12) for arbitrary values of m, n, s, t are considerably more involved, but one can in principle extract the value of $\mathcal{C}_Q$ using Mathematica routines, we have not attempted to do so.

In the table below we list the coefficient of the logarithm and the constant for the gauge exchange diagram E of (4.15).

m	n	s	t	$\mathcal{A}_E$	$\mathcal{C}_E$
m	0	m	0	$-h(m) - \frac{1}{m+1}$	$-(m+1)\sum_{l=0}^{m} \frac{2h(l+1)}{(l+1)^2}(-1)^l\ {}^mC_l$
0	n	0	n	$-h(n) - \frac{1}{n+1}$	$-(n+1)\sum_{l=0}^{n} \frac{2h(l+1)}{(l+1)^2}(-1)^l\ {}^nC_l$
m	0	0	m	$\frac{1}{m} - \frac{1}{m+1}$	$\frac{2}{m^2} - \frac{2}{(m+1)^2}$
0	n	n	0	$\frac{1}{n} - \frac{1}{n+1}$	$\frac{2}{n^2} - \frac{2}{(n+1)^2}$

Table 4: $\mathcal{A}_E$ and $\mathcal{C}_E$ for the gauge exchange E.

To write down the value of the gauge exchange term E for the other case, it is more convenient to consider $E+Q$, where Q is the corresponding quartic contribution. Since the values of the quartic term is known from table 3. the value of E is also known. Below is the table which lists the contribution of $E+Q$ for the remaining cases of m, n, s, t.

m	n	s	t	$\mathcal{A}$	$\mathcal{C}$
m	n	s	0	$\frac{1}{s-m}$	$-\frac{h(m)}{s-m} + {}^sC_m \sum_{l=0}^{m}(-1)^{m-l}\ {}^mC_l \frac{1}{(s-l)^2}$
m	n	0	t	$\frac{1}{t-n}$	$-\frac{h(n)}{t-n} + {}^tC_n \sum_{l=0}^{n}(-1)^{n-l}\ {}^nC_l \frac{1}{(t-l)^2}$
m	0	s	t	$\frac{1}{m-s}$	$-\frac{h(s)}{m-s} + {}^mC_s \sum_{l=0}^{s}(-1)^{s-l}\ {}^sC_l \frac{1}{(m-l)^2}$
0	n	s	t	$\frac{1}{n-t}$	$-\frac{h(t)}{n-t} + {}^nC_t \sum_{l=0}^{s}(-1)^{t-l}\ {}^tC_l \frac{1}{(n-l)^2}$

Table 4: $\mathcal{A}$ and $\mathcal{C}$ for $Q+E$.

If $m \neq s$ the log term for $Q + E$ for arbitrary values of m, n, s, t is given by $1/|m - s|$ and for $m = s$ it is given by $-h(m) - h(n)$. Again we have not listed the values of $\mathcal{C}$ for arbitrary values of the derivatives, but they can be in principle be obtained from (4.15) using routines in Mathematica.

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